

MONTHLY NOTICES
OF THE
ROYAL ASTRONOMICAL SOCIETY

Vol. 103

No. 3

MEETING OF 1943 MAY 14

Professor E. A. Milne, President, in the Chair

The election by the Council of the following Fellows was duly confirmed:—

William Mattock Mason, Airton House, Airton, near Skipton, Yorkshire; and
David C. West, B.Sc., 125 Sherbrooke Street W., Montreal, Canada.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

Polydore Swings, 975 E. Green Street, Pasadena, Cal., U.S.A. (proposed by A. D. Thackeray).

Seventy-five presents were announced as having been received since the last meeting, including—

Hector Macpherson, Guide to the Stars (presented by the Author).

The President welcomed representatives of Polish Institutions and other guests and, after a short introduction, called upon Professor Dingle to give his address on Copernicus, in whose honour the meeting was being held.

AN ANALYSIS OF BRITISH METEOR DATA

J. G. Porter

(Received 1943 March 31)

Summary

1. With a view to a complete statistical analysis of the true paths of meteors, a collection of 1253 accordances has been made, and these have been reduced by a standard method.
2. A study of 61 multiple accordances has given values for the angular errors made by the observers, and preliminary figures for the errors in path, height, magnitude, and speed.
3. A discussion of the previously published paths of Denning and others shows the unsuitable nature of this material for statistical purposes.
4. The improvement due to the technique introduced by Prentice is made apparent.
5. A system of weighting has been evolved for radiants and heights.

Data and Errors

The high percentage of hyperbolic velocities found among meteors by Hoffmeister, Öpik and others has nowhere been more strongly criticised than among amateur observers. In this country, the regular meteor observer, using a technique which can justly be claimed to be superior to any other, has never accepted the "hyperbolic" theory, because he has no direct evidence of it amongst his observations. Criticism of visual observations has been widespread, and often ill informed, and observers have perhaps been deterred from making any firm reply to critics because of lack of knowledge of the errors of observation. The views recently expressed by Plummer*, who stresses the value of visual observations, encourage the belief that a statistical analysis of all available material will throw some light on these problems.

If we except the results of the Arizona Meteor Expedition † (in which the individual paths were admittedly so greatly in error that true paths could not be computed), it is safe to say that no complete systematic analysis of meteor data has ever been made. Numerous partial analyses have appeared, such as the oft-quoted averages of Denning, the attempts at assessing errors by Cookson‡, and the more recent correlations of MacIntosh§ and Plummer||. Yet most of these attempts show one great weakness—they are based on published true paths which form a heterogeneous collection quite unsuitable for statistical purposes. This is true even of Denning's lists, for these were not all computed by a rigid method, but each path was liable to adjustment by the computer exercising an independent and non-methodical judgment. The computer's idea of the probable radiant often exercised a quite disproportionate influence on the resulting path, as will appear later, with the result that the systematic and accidental errors made by the observers were obscured by the non-systematic errors allowed by the computer. Such a condition of things cannot be accepted in a statistical investigation, for a knowledge of the observers' errors is essential, and the computer is only an instru-

* *M.N.*, 101, 82, 1941.

§ *M.N.*, 100, 514, 1940.

† *H.A.*, 105, 30, 1937.

|| *M.N.*, 76, 392, 1916, and 101, 76, 1941.

‡ *M.N.*, 61, 132, 1901.

ment for the attainment of that knowledge. "It is a waste of time to apply the refined theoretical methods of statistics to data which are suspect from the beginning."* It is unfortunate that the contrary view has been expressed by Olivier in a criticism † of the statistical results of the Arizona Meteor Expedition:—

"If there is any department of astronomy in which good judgment on the part of the computer, rather than rigid rules, is necessary, it is in dealing with meteors."

While "good judgment" may be justified in individual cases, it is certain that any serious attempt at a systematic analysis must be based on a random sample of meteor observations reduced by a standard method which is independent of the computer's judgment, and which is capable of assessing weights to all the measurable quantities. With this purpose in view, all available duplicate and multiple observations of meteors seen in this country have been reduced afresh. The multiple observations have then been used to determine the errors of observation, and a system of weighting deduced.

The number of observations available was not as great as had been expected, in spite of the considerable efforts of Mr. J. P. M. Prentice, to whom the writer is greatly indebted for much information. The paths of some 1400 meteors were computed by Denning during his lifetime, but at his death it proved impossible to recover most of the original data, and all that remains to us of his work is one old notebook. The papers of Miss A. Grace Cook and of Mrs. F. Wilson have also been lost with the years, and the following table shows the source and totals of all the accordances that are available:—

The A. S. Herschel letters to Denning, 1877-1900	36	accordances.
B.A.A. Journals and Memoirs, 1890-1914	68	
Denning's notebook, 1895-1900	133	
A. King's notebook, 1898-1920	17	
B.A.A. Observations, 1921-1931	274	
B.A.A. Modern Observations, 1932-1940	725	
	<u>1253</u>	Total.

The total number of observations in this sample is 2669, of which over 90 per cent. were the work of the following 13 first-line observers:—

T. H. Astbury	W. E. Besley	A. S. Herschel
G. E. D. Alcock	H. Corder	A. King
D. Booth	Miss A. Grace Cook	J. P. M. Prentice
T. W. Backhouse	W. F. Denning	Mrs. F. Wilson
J. H. Bridger		

and the work of 174 second-line and casual observers is also included.

These "accordances" are actually observations made by different observers at the same time, and bearing other evidence of being different views of the same meteor. On reduction, a number of these naturally prove to be spurious accordances, and, as a general rule, only about two-thirds of the "accordances" sent to the computer show any real agreement in path.

The modern paths have been reduced by the Computing Section of the B.A.A., and are published from time to time in the *Journal* of that Association. The multiple accordances (Table I) and old accordances (Table IV) have been reduced by the writer, who is also responsible for all the corrections, checking and weighting of all paths, so that the whole sample forms a homogeneous system. Of the 400 old accordances, 258 gave good agreement, representing 65 per cent. of the total. This is roughly the same proportion that is obtained from the whole sample, preliminary results showing that a total of about 800 paths will be available for analysis. This figure, although rather small, is considered

* Yule and Kendall, *Theory of Statistics*, p. 7.

† *Pop. Ast.*, 46, 327, 1938.

satisfactory in view of the accuracy of the results, and the possibility of weighting all the measurable quantities.

It will be seen that the sample represents fairly well the work of the observers during the period 1890–1940.* As will be shown later, the distribution of errors in the sample seems to be normal, but otherwise it is impossible to tell whether this is a typical cross-section of meteor observations. As we have no other information, the lists must be accepted on their face value as a sample dictated more or less by chance. Nevertheless there are two distinct types of observation in the lists, for the old observers quite obviously concentrated on the brighter meteors with the longer paths. There was, moreover, a distinct change in the method of observation, which in the early years seems to have consisted in plotting the meteor paths on a map. It is not known when the use of an extended wand came to be adopted to indicate accurately the direction of the meteor path. It was recommended for use in 1890†, but if used, the increase in accuracy thereby gained was generally lost through failure to read the co-ordinates of position with the attainable precision. The wand (or string) later came to be used to give a “flight-direction,” obtained by extending the meteor path into more suitable star-fields; and by 1929 we find all the British observers quoting co-ordinates for the beginning and end of the flight-direction, together with points on this path which define the actual position of the luminous path of the meteor. The method was improved during the ensuing ten years, principally in the use of cross-bearings, and by the use of distances between known stars, rather than mere guesses of angular distance. All of these changes are due to Prentice, and this investigation indicates that the technique so developed gives superior results to that used in other countries. It is for this reason that this paper deals only with British meteor observations.

It should be made clear that this modern method of observation does not entail the plotting of the paths on a map, for the observer relies solely on his detailed knowledge of the constellations. As an immediate result of this, the modern luminous path is always quoted to tenths of a degree in R.A. and Dec., whereas the old observers usually gave their paths to the nearest degree only. This does not imply that modern observations are ten times as accurate as the old ones, but rather that the data are obtained in such a way as to define more clearly the beginning and ending of the path. The many other improvements introduced by Prentice will be obvious in the analysis that follows, and it is to be hoped that the ill-informed criticism that has been directed against these visual observations will be confined to cases in which the observers are untrained or are using inferior methods.

Method of Reduction.—Of the methods available for the reduction of meteor observations, those which assume that the observations are simultaneous deserve little credit, as they give only an approximate solution of the problem, and that only after the exercise of considerable ingenuity on the part of the computer. Thus Olivier, after describing the Schaeberle method (*Meteors*, ch. xiv), remarks:

“There are many devices which can be employed by the skilled computer, that frequently will permit admissible corrections to be made to the observations from one of the stations. In such cases better results sometimes may be obtained than by using the data as actually furnished.”

This attitude towards the observations first appears in British records in a paper by A. S. Herschel‡ describing the method which he always used, and the descriptions of the artifices which he adopted to get agreement appear in the Meteor Memoirs of the B.A.A.,§

* Although Table IV contains a few earlier paths, effective co-operation in observing dates from the foundation of the B.A.A. Meteor Section in 1890.

† *Mem. B.A.A.*, 1, pt. 2, 20, 1891. Also Denning, *Telescopic Work for Starlight Evenings*, London, 1891.

‡ *M.N.*, 25, 251, 1865.

§ *Mem. B.A.A.*, 10, pt. 1, 1901; 11, pt. 1, 1902; 12, pt. 1, 1903.

and are not calculated to give much confidence in the resulting paths. Denning, on the other hand, consistently used the "earth-point" method, which he has described in his book *Telescopic Work for Starlight Evenings*. The method is quite sound, and can be used to give the individual heights, but it does not seem to have been used in this way. The radiants and azimuths were found on a globe, and the resulting solution carried out by projections on a map. The facility with which these lines could be shifted so as to give better agreement with each other and with the preconceived idea of the radiant, is probably responsible for the continued use of such alterations. Many extravagant examples of this form of adjustment are to be found in the notebooks referred to above, and there is no doubt that the old computers tried to get results from their data at any cost, using the great General Catalogue* of Denning as a guide. Thus most of the meteors in the old lists received names relating to this catalogue. The effect of these adjustments on the radiants and heights may be seen in a comparison of Denning's results with those of the present paper. In this sample it is found that 232 of the paths had been published previously by Denning. Of these 60 must be rejected, either because there is no accordance at all, or else because the results have zero weight. Of the remaining 172 paths, no less than 41 have radiants differing by more than 5° from Denning's, so that some 44 per cent. of Denning's figures must have been adjusted. To an observer of Denning's experience there was, perhaps, some justification in altering a radiant when the angle of convergence of the two apparent paths was quite small, but there is no evidence that the computers restricted themselves to this special case. Thus in the following table, which shows the twelve worst cases of this adjustment, it will be seen that serious alterations have been made even when the angle θ between the paths is quite large. The radiants are given separately, and the differences in height and path are in the sense Denning minus Porter:—

Ref.	G.M.A.T.	Radiant		θ	ΔH_1 km.	ΔH_2 km.	ΔP km.
		Denning	Porter				
M 57	d h m 1896 Mar. 1 08 32	18 + 5	20 - 6	50	+ 6	+ 32	+ 406
	1896 Aug. 6 10 06	42 + 61	6 + 50	13	- 85	- 23	- 78
	1891 Sept. 30 10 02	14 + 7	3 - 1	90	+ 12	- 5	+ 36
	1895 Dec. 11 14 58	50 + 30	74 + 20	35	- 25	- 16	+ 7
M 72	1899 Mar. 1 08 49	119 + 33	133 + 34	43	- 26	+ 5	- 33
	1898 Apr. 18 11 55	274 + 11	285 - 6	14	- 9	- 20	- 28
	1900 May 3 13 58	337 + 0	327 + 6	17	- 16	+ 21	+ 64
	1904 Aug. 11 13 06	45 + 58	4 + 43	16	+ 40	+ 29	+ 18
	1904 Nov. 15 14 40	31 + 20	20 + 15	10	- 11	- 18	- 11
M 76	1914 Apr. 19 09 45	76 + 31	70 + 16	59	- 8	- 8	- 23
	1914 Apr. 20 10 47	235 + 0	267 + 30	10	+ 33	+ 53	- 36
	1913 Aug. 8 11 42	42 + 57	27 + 62	75	- 5	+ 9	+ 38

The differences between the two lists may be expressed in the following means and standard deviations:—

Differences Denning minus Porter:		Frequency = 172	
Height at beginning	+ 3.4 km. ($\sigma = 16.1$)	At end	- 0.8 km. ($\sigma = 11.5$)
Length of path	+ 14.8 km. ($\sigma = 42.7$)	Speed	+ 6.2 per cent. ($\sigma = 28.1$)

The differences in height and path length show greater deviations than the standard errors in Table II; the speeds are not strictly comparable in this way, since many of the old accordances had only one estimate of duration whereas the speeds in the list of multiple accordances are all based on at least two estimates. However, there is sufficient

* Mem. R.A.S., 53, 203, 1899.

evidence here of the unstable nature of Denning's published paths, and of their heterogeneous quality, quite unsuited to statistical analysis.

It was not until 1930 that the protests of Prentice had some effect, and the practice of adjusting the observations ceased to a large extent. There is, however, definite evidence of the existence of the method among the results given by King in the last Meteor Memoir of the B.A.A.* In 1936 Davidson produced his elegant method † for meteor reduction, using co-ordinate geometry. A detailed account of this method, and of the least squares solution for multiple observations which has been derived from it, have appeared elsewhere. ‡ It will be sufficient to say that the method finds the point of intersection of one of the observations with the plane of the observations of the other observer, so that no assumptions of simultaneity are made, and the method has proved ideally suited to the work. The only alterations that have been made are such as to make the calculation of angular errors possible.

The Multiple Errors.—Included in the sample are 100 multiple accordances, of which 61 gave satisfactory results on reduction. In most of the other 39 cases, the erroneous path proved to be that of a casual or second-line observer, and it is the experience of the writer that the work of such observers is so far below the standard of experienced workers as to be of little value. In these cases, therefore, the erroneous path has been rejected and the remaining paths computed as a duplicate. The results for the 61 multiples are given in Tables I and IA, and the errors made by the observers are given in Table II as means and standard deviations. It had been hoped to deduce the errors made by individual observers, so that weights could be given to each, but this proved impossible owing to the small numbers available. The observations have therefore been divided into "Old" and "New," the latter being the modern type of observation using the improved technique referred to above. The date at which the change occurred may be taken, for the purpose of this sample, as 1931, and the improvement in the results is shown clearly in the standard deviations of the various quantities.

The Angular Errors.—The radiant errors are merely the residuals obtained in the observational equations, the positive sign indicating that the observed path passes N. of the correct radiant. The actual angular errors made by the observers may be considered to consist of *sliding errors* ΔE along the path (positive in the direction of motion of the meteor), and *offset errors* Δ perpendicular to the path (positive if N. of the true path). These errors are computed in circular measure, but are given in Table II in degrees.

The offset errors Δ_1 and Δ_2 were computed from the direction cosines (l, m, n) of the observed point on the path, and the equation for the corrected plane of observation, written in the form

$$Ux + Vy + Wz = 0.$$

Since Δ is always small, we may write

$$Ul + Vm + Wn = \Delta.$$

The resulting values of Δ_1 and Δ_2 are readily checked against ΔR .

The sliding errors ΔE_1 and ΔE_2 should logically be computed by a separate least squares solution, but it was decided to use the simpler method of weighting the heights in such a way as to make the algebraical sum of the sliding errors equal to zero at each end of the path. As the errors are quite large, this method is satisfactory and is easily applied. If the path is of length p and angular length π then the rate of change $d\pi/dp$ for any observation may be regarded as its appropriate weight. This quantity is equal to $\sin E/r$ where E is the angular distance of the observed point from the radiant and r is the radius vector. This weight having been used to obtain the weighted mean heights, it is a simple matter to use it again to find the angular error.

The twist τ in the path is a simple guide to the accuracy of its direction, and has been computed from the angle between the observed and computed planes of observation, without reference to the actual point of intersection of these planes. The twist is checked against the offset errors and the angular length of path.

* Mem. B.A.A., 32, pt. 1, 1936. † J.B.A.A., 46, 292, 1936. ‡ Mem. B.A.A., 34, pt. 4, 1942.

TABLE I
True Paths from Multiple Observations

No.	Observed Radiant	Cor- rected Radiant	ΔR	a	ϵ	M	ΔM	H_1	H_2	P	S	V	ΔH_1	ΔH_2	ΔT	Refs.
1	113.5 + 36.5	115 + 36	2.2	52	63	-0.3	0.8	97	52	57	...	47	4.7	3.4	...	M 57
2	327.8 + 57.9	332 + 56	1.7	27	100	-1	...	66	34	69	23	27	3.6	1.2	...	"
3	263.6 + 25.9	265 + 25	1.2	31	58	-1.7	1.3	94	67	52	...	51	2.7	4.2	...	"
4	258.2 - 25.4	260 - 29	2.0	7	89	-6	...	74	69	40	10	32	0.6	0.0	...	"
5	330.4 + 39.0	331 + 39	1.6	65	67	-2.1	1.1	108	84	26	...	45	9.2	7.3	...	"
6	149.9 + 24.8	150 + 25	0.1	63	12	2.3	0.4	113	77	38	...	72	8.8	15.3	...	"
7	339.2 - 21.3	340 - 23	1.6	7	70	2.2	0.6	90	85	46	...	43	0.8	0.3	...	M 72
8	341.3 - 10.4	343 - 12	0.9	5	71	-3.5	...	62	54	94	19	43	0.1	0.9	12	"
9	45.1 + 60.1	47 + 60	0.5	33	42	-3.5	1.8	138	102	64	...	60	2.9	3.3	...	"
10	281.3 - 19.1	282 - 22	1.8	19	92	1.1	0.8	76	71	17	...	31	4.6	2.9	...	"
11	39.0 + 57.5	41 + 58	1.6	34	41	1.2	0.9	154	121	57	71	61	3.6	2.7	...	"
12	41.6 + 52.8	43 + 53	1.9	28	35	-3.6	0.8	96	66	61	36	64	7.5	4.6	1	"
13	42.9 + 57.5	44 + 58	1.7	37	40	-3.9	2.0	170	85	139	104	61	2.4	7.2	...	"
14	40.7 + 55.8	42 + 56	2.3	37	38	0.2	0.3	125	96	48	68	62	2.9	0.4	...	"
15	266.2 + 47.4	268 + 47	0.5	37	74	0.8	0.3	197	113	136	147	40	0.7	5.8	26	"
16	246.0 - 32.0	240 - 44	2.5	-1	140	-6	0.6	89	90	100	33	19	0.2	0.2	...	M 72
17	271.4 + 29.3	273 + 28	2.5	21	56	-2.2	0.7	119	81	95	54	52	6.2	4.1	...	"
18	313.0 + 34.3	314 + 34	0.3	56	65	v.b.	...	170	121	61	...	46	12.2	13.1	...	"
19	273.9 + 38.2	276 + 36	1.5	2	76	-5.0	0.7	83	66	246	36	40	1.9	0.4	23	"
20	150.3 + 22.5	151 + 22	1.2	61	9	-2.5	0.3	129	96	37	64	72	3.3	6.4	47	"
21	352.8 + 40.6	354 + 41	1.9	66	87	1.0	0.2	95	50	48	21	33	5.2	5.1	...	M 76
22	66.5 + 42.2	68 + 42	0.2	14	32	-0.3	1.1	126	109	60	...	65	5.1	4.2	...	"
23	85.5 + 41.4	87 + 41	1.5	21	24	v.b.	...	99	86	35	12	68	4.5	4.5	...	J 32
24	114.7 + 9.8	113 + 6	0.8	44	123	-6	...	98	43	79	16	21	10.2	15.8	52	B 14
25	36.3 + 22.7	37 + 22	1.7	15	13	-4.6	2.5	140	113	103	49	71	6.2	3.0	61	B 14
26	285.6 + 26.8	281 + 22	1.0	32	129	-0.3	0.1	78	55	42	15	20	3.7	6.4	70	"
27	137.8 + 41.4	138 + 40	2.3	3	23	0.2	0.8	133	128	67	56	69	0.2	0.4	17	"
28	51.0 + 58.5	53 + 58	3.4	34	44	-0.3	0.9	107	95	22	40	59	0.1	1.2	39	"
29	36.2 + 55.2	37 + 55	0.4	45	39	-1.1	1.0	116	89	37	29	62	2.1	3.4	13	"
30	312.0 + 39.7	308 + 36	3.8	36	123	-2.3	1.6	79	63	27	7	21	4.4	4.7	40	B 15

TABLE IA

No.	☉	Date (G.M.A.T.)	No.	☉	Date (G.M.A.T.)	No.	☉	Date (G.M.A.T.)
		d h m			d h m			d h m
1	262.3	1892 Dec. 12 10 52	11	137.7	1899 Aug. 9 10 19	21	189.8	1912 Oct. 2 08 40
2	351.1	1895 Mar. 10 14 27	12	138.7	Aug. 10 10 14	22	136.3	1913 Aug. 8 10 54
3	30.4	Apr. 19 10 59	13	"	Aug. 10 10 55	23	193.5	1921 Oct. 6 09 28
4	80.5	June 10 09 59	14	"	Aug. 10 11 02	24	328.9	1922 Feb. 17 11 32
5	135.9	Aug. 7 11. 04	15	32.0	1900 Apr. 21 10 05	25	142.7	Aug. 15 10 25
6	234.1	1896 Nov. 14 17 45	16	122.2	July 24 10 49	26	220.8	1923 Nov. 3 08 59
7	131.6	1897 Aug. 2 10 52	17	31.8	1901 Apr. 21 10 08	27	221.7	Nov. 4 06 10
8	139.9	1898 Aug. 11 08 58	18	111.2	1902 July 13 10 30	28	130.7	1924 Aug. 2 11 22
9	"	Aug. 11 10 16	19	308.6	1903 Jan. 28 11 44	29	133.6	Aug. 5 12 15
10	103.4	1899 July 4 10 50	20	233.4	Nov. 15 18 08	30	208.2	1930 Oct. 21 12 03

TABLE I—continued

No.	Observed Radiant	Cor- rected Radiant	ΔR	a	ϵ	M	ΔM	H_1	H_2	P	S	V	ΔH_1	ΔH_2	ΔT	Refs.
31	94.3 + 5.5	95 + 5	0.5	32	30	0.8	0.7	123	101	43	69	66	1.0	6.0	4	B 15
32	109.2 + 21.7	109 + 22	0.3	41	16	-0.1	1.0	117	100	27	46	71	1.8	8.3	34	"
33	110.4 + 31.6	111 + 31	0.9	60	18	1.1	0.6	155	100	55	134	70	10.7	5.2	27	"
34	62.1 + 21.7	62 + 21	0.1	57	81	-6	1.2	132	62	82	37	37	13.6	9.1	21	"
35	154.1 + 22.4	154 + 22	1.2	52	11	-1.4	0.5	130	82	59	63	72	4.6	2.2	6	"
36	269.6 + 36.0	270 + 36	0.2	38	62	-0.9	0.4	126	63	101	208	48	9.4	5.1	30	"
37	334.1 + 31.0	335 + 31	0.4	51	69	1.4	1.4	94	84	12	54	44	1.9	4.4	10	"
38	90.4 + 43.8	90 + 44	1.8	80	57	1.6	1.2	68	55	13	16	51	5.6	5.8	20	"
39	148.3 + 21.9	149 + 22	0.1	31	9	-3.7	2.2	127	93	66	68	72	10.4	1.8	51	"
40	126.7 + 26.5	127 + 26	1.1	39	67	1.8	0.5	77	64	20	26	45	1.5	3.7	13	B 15
41	351.6 + 14.1	352 + 14	1.0	19	20	-2.3	1.7	114	94	59	48	70	3.0	5.0	24	
42	20.0 + 26.0	21 + 26	0.3	16	24	-1.5	1.2	113	95	62	62	68	3.6	0.4	14	
43	28.3 + 11.4	29 + 11	0.9	28	18	2.3	0.5	120	101	40	45	70	4.2	1.7	31	
44	51.9 + 49.6	53 + 50	2.5	21	33	-0.6	1.1	109	97	30	34	65	3.8	0.2	43	
45	44.5 + 58.6	46 + 59	0.7	33	41	-4.0	0.5	100	79	39	45	61	2.8	2.6	...	
46	53.6 + 57.8	55 + 58	2.3	30	41	-1.7	1.3	108	91	33	39	60	2.5	4.6	0	
47	44.8 + 57.2	46 + 57	0.4	35	39	-4.3	1.7	107	83	41	49	62	7.8	1.7	42	
48	47.2 + 44.0	48 + 44	0.1	24	26	-5	1.3	103	85	44	40	68	1.7	0.8	36	
49	65.0 + 47.3	66 + 47	1.0	21	33	0.8	1.1	104	94	24	45	65	1.3	0.9	10	
50	48.3 + 53.8	49 + 54	0.8	33	36	-6	0.6	114	82	58	52	63	7.6	3.1	22	
51	49.3 + 51.9	50 + 52	1.2	32	34	1.2	1.5	113	100	24	42	64	0.9	1.1	8	
52	33.3 + 59.8	34 + 60	0.9	50	43	2.1	0.5	107	88	24	37	60	4.7	5.1	18	
53	37.4 + 54.5	38 + 55	0.0	47	38	-3	0.0	144	97	64	116	62	10.6	9.1	19	
54	50.8 + 56.3	52 + 56	2.0	35	38	-4.2	1.8	139	78	103	63	62	14.3	6.4	16	
55	327.3 + 17.2	328 + 17	0.3	42	74	2.6	0.6	97	80	25	28	41	5.9	2.4	23	
56	56.8 + 58.1	58 + 58	0.2	36	42	2.2	0.7	102	91	19	48	60	1.2	2.1	28	
57	44.8 + 55.3	45 + 55	0.4	44	38	2.6	0.4	112	98	20	41	62	7.3	2.2	56	
58	296.4 - 24.5	297 - 32	0.8	13	122	1.6	0.4	79	71	35	12	21	2.2	0.7	17	
59	278.4 + 38.6	279 + 38	0.1	41	65	1.7	0.7	106	98	12	17	46	0.9	0.9	8	
60	346.5 + 71.8	348 + 72	2.3	59	64	1.1	0.3	97	82	17	25	47	2.3	3.9	30	
61	37.5 + 38.0	38 + 38	0.2	49	22	-0.8	1.1	113	92	28	45	69	1.2	5.3	26	

TABLE IA—continued

No.	☉	Date (G.M.A.T.)	No.	☉	Date (G.M.A.T.)	No.	☉	Date (G.M.A.T.)
31	208.2	1930 Oct. 21 13 23	41	101.1	1932 July 2 11 30	51	139.3	1932 Aug. 11 11 08
32	212.2	Oct. 25 13 39	42	"	July 2 12 10	52	"	Aug. 11 11 59
33	213.3	Oct. 26 14 55	43	138.4	Aug. 10 12 32	53	"	Aug. 11 12 04
34	234.2	Nov. 16 12 47	44	139.2	Aug. 11 09 48	54	141.2	Aug. 13 10 57
35	234.4	Nov. 16 16 23	45	"	Aug. 11 10 01	55	137.5	1939 Aug. 10 09 48
36	23.4	1931 Apr. 13 11 38	46	139.2	Aug. 11 10 26	56	138.5	Aug. 11 11 29
37	142.4	Aug. 15 09 40	47	"	Aug. 11 10 31	57	"	Aug. 11 12 07
38	234.0	Nov. 16 13 35	48	"	Aug. 11 10 39	58	"	Aug. 18 09 16
39	"	Nov. 16 13 38	49	139.3	Aug. 11 10 58	59	145.2	1940 May 4 11 04
40	279.5	Dec. 31 09 48	50	"	Aug. 11 11 03	60	137.3	Aug. 9 10 19
						61	138.4	Aug. 10 13 26

The quantities tabulated are as follows:—

The date is given in G.M.A.T. (day begins at noon) as generally given by the observers, and for convenience in comparing old observations.

☉ *the Sun's true longitude*, referred to the equinox of 1950-0, and used as a basis of time measurement. *Radiant* in Table I gives the results of the least-squares solution, and corresponds to the radiant given by Denning, which were always uncorrected.

Corrected radiant. Corrections for diurnal aberration and zenithal attraction have been applied in all cases, the resulting radiant corrected for precession to 1950-0, and rounded off to the nearest degree in R.A. and Dec.

Wt. gives the weight assigned to a duplicate radiant on a scale of 5, according to the system explained below.

ΔR *the radiant error* is the perpendicular distance from the radiant to the observed flight-direction. The R.M.S. value is given in Table I, and corresponds to the radius of the circle of error as used by the old computers.

a *the altitude of the radiant*, as seen from the end of the path.

ϵ *the elongation of the corrected radiant* from the apex of the Earth's way.

M *the mean zenithal magnitude of the meteor* as it would be seen if in the zenith at a height of 100 km.

The method of computing this quantity is given in *Mem. B.A.A.*, 34, part 4, 45, 1942.

ΔM *the mean deviation in magnitude*.

H_1, H_2 *the weighted mean heights* in km. at beginning and end of path respectively. The method of weighting is explained below.

$\Delta H_1, \Delta H_2$ *the weighted mean deviations in height*.

P *the weighted length of path* in km.

S *the harmonic mean speed* of the individual estimates in km./sec.

ΔT *the mean deviation in speed*, expressed as a percentage of the observed speed. May also be regarded as the mean error in estimate of duration, as a percentage of the true time of flight.

V *the theoretical parabolic velocity* of the meteor relative to the Earth, computed from ϵ .

Ref. gives a reference to any previous publication of the path. These are abbreviated as follows:—

M 57 = *M.N.*, 57, 164, 1897

M 72 = *M.N.*, 72, 426, 1912

M 76 = *M.N.*, 76, 224, 1916

J 32 = *J.B.A.A.*, vol. 32

O 46 = *Obs.*, vol. 46

and similarly.

B 1 = *Mem. B.A.A.*, 1, pt. 2, 1891

B 8 = *Mem. B.A.A.*, 9, pt. 1, 1900

B 9 = *Mem. B.A.A.*, 10, pt. 1, 1901

B 11 = *Mem. B.A.A.*, 12, pt. 1, 1903

B 14 = *Mem. B.A.A.*, 24, pt. 3, 1923

B 15 = *Mem. B.A.A.*, 32, pt. 1, 1936.

Rejections.—A preliminary investigation showed that the offset errors were an excellent guide to the value of an accordance, the modern observations giving small values of Δ_1 and Δ_2 of the same sign, showing that the path had been moved as a whole, parallel to itself. The R.M.S. errors were in the neighbourhood of 0.017 radians, and accordingly 0.060 radians was taken as the upper limit, and all observations exceeding this value were rejected. In the same way, a value of 0.160 radians was taken for the upper limit of ΔE when weighting the heights, and values in excess of this were not used. The same value has been used in rejecting duplicate observations.

Discussion of the Errors

It will be noticed that the means in Table II (with the exception of the heights) all have values approximating to zero, as would be expected from the method of calculation. In each case, the distribution proved to be fairly normal, and the presence of systematic errors was never in evidence. For the sake of completeness, the average and median errors (taken without regard to sign) have been computed, and are tabulated in Table III. Thus the dispersion of the errors may be seen from the s.d.'s, while the figures generally indicate the normal distribution. The medians are probably the most useful of these results, and indicate clearly the great improvement effected by the technique developed by Prentice. The standard error (or probable error) of the mean, has not been given, as being of little interest, but may be calculated approximately from the figures given.

TABLE II
Mean Errors and Standard Deviations

Error		120 Old Observations		68 New Observations		All Observations	
		Mean	s.d.	Mean	s.d.	Mean	s.d.
ΔR	Radiant	-0.06	1.62	-0.07	1.13	-0.06	1.47
τ	Twist	-0.26	3.07	-0.19	2.27	-0.22	2.81
Δ_1	Offset (beginning)	+0.06	1.10	0.00	0.65	+0.03	0.96
Δ_2	Offset (end)	+0.10	1.28	+0.05	0.81	+0.09	1.13
ΔE_1	Sliding error (beginning)	-0.28	3.30	+0.09	2.97	-0.15	3.19
ΔE_2	Sliding error (end)	+0.04	3.45	-0.06	2.60	0.00	3.16
$\Delta \pi$	Angular path	0.00	3.80	-0.16	3.29	-0.06	3.61
Δp Path length km.		+3.7	21.1	+0.6	12.3	+2.5	18.2
ΔH_1	Height (beginning) km.	4.6	3.5	4.2	3.3	4.4	3.5
ΔH_2	Height (end) km.	4.6	3.8	2.9	2.2	4.0	3.4
ΔT	Speed per cent.	-4.8	38.2	0.0	30.6	-2.1	34.3
ΔM	Magnitude	0.0	1.30	-0.5	1.06	0.0	1.25

TABLE III

Error		Averages			Medians		
		Old	New	Total	Old	New	Total
ΔR	Radiant	1.23	0.78	1.07	1.0	0.45	0.7
τ	Twist	2.47	1.60	2.15	2.1	1.1	1.7
Δ_1	Offset (beginning)	0.86	0.49	0.73	0.7	0.35	0.6
Δ_2	Offset (end)	0.95	0.57	0.81	0.7	0.35	0.6
ΔE_1	Sliding error (beginning)	2.46	2.32	2.41	1.8	1.7	1.7
ΔE_2	Sliding error (end)	2.71	2.04	2.46	2.2	1.5	1.7
ΔT	Speed per cent.	29.9	24.9	27.1	29	21.5	23
ΔM	Magnitude	0.98	0.91	0.95	0.75	0.75	0.75

Errors in Radiant.—In a multiple observation the radiant error is obtained in the course of a least squares solution, but in the case of a duplicate observation there is no such method of judging the error. In this case the error depends not only on the twist in the path, but also on the angle between the paths, and the angular distance of the path from the radiant. It has often been stated that a much foreshortened path leads to an accurate radiant, although the heights calculated from it may be in error. It is easily seen that this is not always the case, for the error due to the twist in the path will be proportional to the *sine* of the distance from the radiant, and to the *cosecant* of the angle between the paths. Thus the effect of the angle between the paths is much the greater, and this fact may be demonstrated from the sample. Each multiple observation provides a number of pairs of paths which intersect in pseudo-radiants, the distance of which from the actual radiant may be computed. In this way 201 pseudo-radiants have been computed from the sample, and the distances from the actual radiant and the angles between the pairs of paths computed also. These distances and angles are

found to range from 0° to 90° , so that some form of weighting is clearly necessary. The correlation of (distance)/(mean angular distance from the actual radiant) gives $r = +0.13$, while for (distance)/(angle between paths), $r = -0.50$. In this latter case the regression is not linear (as was to be expected, since the cosecant of the angle is involved) and the correlation ratio for distance on angle between paths is 0.79. It was not thought necessary to study this particular correlation further, since the only requirement was a simple method of weighting the radiant, and this seems to be provided by the angle between the paths, a quantity which is very easily calculated from the data in Davidson's method. A smooth curve of regression was therefore drawn from the means of the angle-arrays in the contingency table, and the weights and anticipated errors derived from the graph are shown in the following table, together with the actual mean errors for each group in the sample:—

Weight	Angle	Anticipated Error	Mean Error from Sample	Frequency
0	$0^\circ-15^\circ$	$> 6^\circ$	32.6	36
1	$15^\circ-30^\circ$	$4^\circ-6^\circ$	5.7	36
2	$30^\circ-40^\circ$	$3^\circ-4^\circ$	3.7	25
3	$40^\circ-50^\circ$	$2\frac{1}{2}^\circ-3^\circ$	1.8	32
4	$50^\circ-60^\circ$	$2^\circ-2\frac{1}{2}^\circ$	2.4	23
5	$> 60^\circ$	$< 2^\circ$	1.7	49
				<u>201</u>

The agreement is sufficiently good, and in only one group does the actual mean fall outside the anticipated range. The rejection of meteors having an angle between the paths less than 15° means that about one-sixth of the meteors in this sample would have to be rejected, but this figure will vary with the sample. Thus in the list of old duplicates in Table IV only 28 (7 per cent.) have been so rejected. The rejections from the sample include most of the abnormal errors that arise, but a few of these large errors are left in the remaining 165 pairs, and this suggests the use of the median as a better guide to the value of the radiants than the A.M. The median of this sample is $2^\circ.06$, so that it may be anticipated that 50 per cent. of all duplicate observations will give radiants within 2° of the truth. In the case of multiple observations, a value well below a half of this may be expected, as may be seen from Tables II and III. These values apply only to accordances by trained observers.

The Offset Errors.—These are satisfactorily small, and are usually of the same sign, in so far as modern observations are concerned. There is nothing to indicate that these errors (and therefore the twist in the path) are other than accidental errors, since there appears to be no correlation between them and any other measurable quantity. A suggestion was made by Prentice that these errors might arise as an effect of parallax as between the two eyes and the string, and would be most serious with a meteor whose apparent path is vertical. This idea was tested by computing the apparent angle of descent of all meteors in the sample and correlating with the twist in the path. The value $r = -0.052$ was obtained, so that this theory is not supported by the evidence. There seems no reason to doubt that the errors arise from faulty estimates of distance on the part of the observer. Thus a flight direction recorded as

$$1^\circ \text{ n.p. } \psi \text{ Pegs, } \frac{1}{3}^\circ \text{ s.f. } 74 \text{ Pegs, } 1\frac{1}{2}^\circ \text{ s.f. } 70 \text{ Pegs,}$$

may easily have small fractional errors in each of the angles. In this case (an actual record) the centre point, for example, is $0^\circ.4$ from the great circle joining the other two.

The Sliding Errors.—These seem to be very large, for a number of observations have been rejected, so that the values in Tables II and III are in the nature of lower limits. It will be noticed that, although Prentice's new technique has halved the twist, the radiant and offset errors, there has been little improvement in the sliding errors.

This is due to the fact that the modern observers have concentrated on the attainment of accuracy in the radiant, at the expense of accurate positioning of the path.

There appears to be no justification for the old belief that the end of a path is better observed than the beginning. This is what would be expected, and seems to be proved by the fact that the errors in height are greater at the beginning than at the end. But this last fact is caused by the nature of the problem, since equal sliding errors at the beginning and end of path must give greater deviations in height at the beginning because of the foreshortening of the path.

Again, these sliding errors do not seem to be due to sliding the path *as a whole*, for if this were the case the values of $\Delta E_2 - \Delta E_1$ should all approximate to zero. Actually the s.d.'s of this quantity, which are given in Table II as the angular path error $\Delta\pi$, are as great as those of ΔE_1 and ΔE_2 .

An attempt has been made, without much success, to correlate the sliding errors with the other measured quantities. A value of $r = +0.42$ was found for the correlation sliding error/time of flight. There seems at the moment no explanation for this extraordinary result, which is quite the opposite of what was expected, and further data on this point are being collected.

The Magnitudes.—The error in magnitude is disappointing, and is due to the fact that most of the observers merely guess at the magnitude of the meteor. It was not until 1939 that Prentice introduced the method of comparison magnitudes, and the result of this can only be inferred at present from the few comparison figures available from these multiples. The 12 comparison magnitudes have a mean error of $+0.15$ ($\sigma = 0.49$) for observers GEDA and JPMP, while these same observers had for their previous 41 observations a mean of -0.23 ($\sigma = 1.14$). This would appear to be a big improvement, but the frequency is too small to form a sound judgment.

Prentice has suggested that there will be an error in magnitude determination due to the foreshortening of the meteor. Thus a meteor coming directly towards an observer, and so appearing stationary to him, should also appear of different brightness from one travelling along a path at right angles to him. If this effect does exist, there should be a definite correlation between the magnitude error and the angular distance of the meteor from the radiant, but examination of this point gives $r = +0.017$. The effect, if present, is evidently completely overshadowed by the other errors of observation.

Speeds.—It seems to have been the practice with the old computers to obtain the mean speed by dividing the mean path by the mean duration of flight. This method would be satisfactory if both observers saw the whole of the path, although even so, the method had the effect of giving the greatest weight to the slowest speed. For the mean speed will be given as

$$\frac{P_1 + P_2}{T_1 + T_2} = \frac{S_1 T_1 + S_2 T_2}{T_1 + T_2},$$

so that the greatest weight is given to the speed associated with the longer time of flight. This method would give speeds that were too low, were it not for the fact, as shown below, that the old paths were generally too long.

If the observer's estimate of duration is always that for the whole path, then there should be no correlation at all between the errors in path length and the errors in timing, since both sets of errors should be entirely at random. On correlating the path errors with the timing errors (computed as deviations from the mean time of flight, in order to avoid any spurious correlation) a value of $r = +0.43$ was obtained. As the numbers were small, and the presence of random large errors may affect the result, a number of duplicate results were added. This gave a frequency of 803, and $r = +0.436$. This seems to justify the conclusion that there is a definite correlation between path errors and timing errors, and that if the observer sees only part of the path, his estimate of the duration relates to the part seen. The individual paths and durations should therefore

be used to obtain the individual speeds, and hence the harmonic mean speed is the best value to employ. This is the value given in Tables I and IV. It is difficult to express the mean deviation in speeds when the mean is harmonic, so that the error in timing has been expressed as a percentage of the true time. This error is probably a combination of the errors in time and path, but as these errors are large, they are not easily separated.

The Heights.—There is a certain amount of evidence in the notes examined that the old computers were in the habit of taking the greatest computed height at the beginning, and the least height at the end of the path. This practice is hardly consistent with the belief that both observers see the same path, nor yet with the method of computing mean speeds! It is clear that the method gave paths that were too long, and speeds that were too high, and this is evident in the comparison figures Denning minus Porter given above. Other workers (*e.g.* King) in later years used mean heights, and this variation in method is yet another factor that makes the published paths unsuitable for statistical purposes. The method that has been used in this paper gives results which differ little from the arithmetic mean, and it is recommended that for general work, where the angular errors are not required, the A.M. should be used, as involving less laborious computation. The differences between the A.M. and the equal-angle mean used in this paper have been calculated for each observation in this sample, and have the values

at beginning of path	+0.11 ($\sigma=2.37$) km.
at end of path	+0.34 ($\sigma=1.65$) km.

These means and s.d.'s fall well within the limits given in Table II, so that the A.M. is quite suitable for most purposes.

The means of the errors in height given in Table II are the weighted means, and are not distributed normally, since the signs have not been used. The medians in such a skew distribution may be considered more reliable, and these come to 3.6 km. at the beginning and 3.4 km. at the end of path (using all meteors in Table I). Thus the heights may be considered to be computed to within 4 km., but the range over which errors may be allowed must be considerably extended beyond the limits used at present.

The discussion of sliding errors, heights, magnitudes and speeds will be completed when the greater numbers available on reduction of all the duplicate observations can be used.

TABLE IV
True Paths from Old Duplicate Observations

No.	Cor- rected Radiant	Wt.	α	ϵ	M	ΔM	H_1	H_2	P	S	V	ΔH_1	ΔH_2	ΔT	Ref.
	$^{\circ}$		$^{\circ}$	$^{\circ}$										%	
1	74+42	5	14	125	-6	...	86	56	120	...	21	2.7	1.3	...	M 25
2	28+57	1	23	75	-5	0.9	137	96	102	62	40	1.3	7.1	...	
3	55+55	2	29	39	-4	1.1	94	61	67	80	62	0.4	0.6	14	
4	72+62	1	33	49	1.3	0.4	98	66	58	140	56	4.7	1.9	...	
5	43+52	2	30	34	-4	1.3	143	98	86	47	64	11.2	0.8	...	
6	56+65	1	38	48	-1.0	0.3	184	122	99	220	57	5.1	3.7	26	
7	43+56	5	42	38	v.b.	...	113	82	45	...	62	5.6	3.8	...	M 57
8	150+25	2	59	12	v.b.	...	122	90	38	...	72	0.8	10.5	...	"
9	217-15	5	26	119	...	...	62	39	52	...	22	11.6	2.7	...	"
10	88+18	1	47	58	2.9	0.2	137	108	41	...	51	14.6	5.0	...	B 1
11	59+32	5	14	43	-1.9	0.2	125	85	165	...	60	21.4	1.7	...	M 57
12	3-1	5	35	97	-5	0.4	75	44	54	18	29	11.0	0.5	51	"
13	233+55	4	22	69	1.2	0.1	99	73	64	...	44	2.2	5.3	...	"
14	65-20	1	18	128	0.3	0.4	93	76	54	15	20	3.8	2.2	...	"
15	353+61	1	30	112	-2.5	...	117	80	70	...	24	2.5	6.3	...	"
16	306+30	1	12	52	-0.6	0.1	138	109	136	82	54	0.1	7.6	...	M 57
17	34+61	1	44	45	1.0	0.3	181	95	122	...	59	13.2	5.9	...	"
18	49+60	5	32	42	3.4	0.2	120	98	40	61	60	4.6	2.7	...	"
19	37+57	3	39	40	0.4	0.1	148	108	63	56	61	27.6	15.9	...	"
20	33+52	4	44	37	-3.5	0.1	114	85	42	29	63	0.2	1.1	...	"
21	318-19	5	29	171	1.5	...	73	44	58	...	17	3.3	1.3	...	M 57
22	74+20	2	44	96	-3	1.5	110	45	94	...	29	38.6	7.9	...	"
23	20-6	4	4	123	0	1.3	83	57	246	17	21	2.9	7.2	53	"
24	196-21	1	7	86	-4	1.0	82	55	199	16	34	...	1.5	...	"
25	41+55	1	31	37	0.7	1.0	105	79	47	...	63	9.1	3.7	...	"
26	66+18	5	50	76	-1.2	0.2	123	88	46	25	39	13.8	3.0	...	M 57
27	139+34	4	63	21	-3.6	1.9	147	109	43	96	69	1.9	5.9	...	"
28	156+24	1	34	13	-3	0.7	141	89	94	102	71	2.5	9.4	...	"
29	151+24	2	61	11	-1	1.3	143	108	41	...	72	2.5	1.2	...	"
30	128+31	1	67	25	2.7	0.1	82	70	13	...	68	8.3	8.1	...	"

TABLE IVA

No.	$\odot$	Date (G.M.A.T.)	No.	$\odot$	Date (G.M.A.T.)	No.	$\odot$	Date (G.M.A.T.)
	$^{\circ}$	d h m		$^{\circ}$	d h m		$^{\circ}$	d h m
1	40.9	1865 Apr. 29 12 42	11	112.1	1891 July 13 12 54	21	232.9	1895 Nov. 14 06 20
2	70.6	1877 May 30 11 26	12	188.3	Sept. 30 10 02	22	260.6	Dec. 11 14 58
3	136.5	Aug. 7 10 54	13	282.0	1895 Jan. 1 06 43	23	342.6	1896 Mar. 1 08 32
4	136.5	Aug. 7 11 44	14	282.0	Jan. 1 07 23	24	20.3	Apr. 8 08 20
5	139.4	Aug. 10 10 25	15	351.1	Mar. 10 15 24	25	139.5	Aug. 10 10 07
6	139.4	1877 Aug. 10 10 45	16	25.5	1895 Apr. 14 11 44	26	232.0	1896 Nov. 12 15 04
7	142.4	1888 Aug. 13 11 33	17	135.9	Aug. 7 11 28	27	232.0	Nov. 12 15 44
8	233.2	Nov. 13 17 19	18	139.7	Aug. 11 10 07	28	234.0	Nov. 14 14 28
9	69.5	1889 May 29 10 44	19	139.8	Aug. 11 10 58	29	234.1	Nov. 14 17 27
10	235.5	1890 Nov. 16 12 15	20	140.7	Aug. 11 11 42	30	234.2	Nov. 14 18 05

TABLE IV—continued

No.	Cor- rected Radiant	Wt.	<i>a</i>	<i>ε</i>	<i>M</i>	ΔM	<i>H</i> ₁	<i>H</i> ₂	<i>P</i>	<i>S</i>	<i>V</i>	ΔH_1	ΔH_2	ΔT	Ref.
	[°] 212 - 2		[°] 39	[°] 93											
31	212 - 2	5	39	93	1.6	0.0	87	67	32	29	30	5.6	4.4	...	M 72
32	194 + 1	5	42	111	1.6	0.4	92	79	19	33	24	1.4	1.9	...	"
33	222 - 5	3	36	94	1.1	0.8	101	74	47	...	30	1.9	2.5	...	"
34	297 - 3	2	16	24	2.5	0.3	110	100	40	59	68	3.9	5.7	...	"
35	296 - 9	2	31	105	1.0	0.6	79	72	13	21	26	5.5	3.7	...	"
36	112 + 67	1	29	69	2.4	0.1	76	73	6	...	44	1.3	2.2	...	M 72
37	311 - 2	5	35	89	-6	1.2	119	84	61	66	32	11.8	0.0	...	"
38	52 + 46	2	13	30	-7	0.5	206	186	82	50	66	5.4	3.3	...	"
39	49 + 55	5	51	38	-0.1	0.1	215	120	123	146	62	10.2	2.7	...	"
40	55 + 60	3	52	43	1.3	0.4	192	139	68	...	60	3.9	5.3	...	"
41	45 + 58	5	57	31	-4	2.8	175	133	50	...	66	37	3.6	...	M 72
42	48 + 55	5	35	37	0.6	0.4	115	89	45	...	63	8.2	4.1	...	"
43	119 - 8	3	27	148	v.b.	...	194	49	311	18	18	6.2	3.8	11	"
44	262 + 13	3	10	50	1.0	1.2	116	102	84	...	56	4.3	2.4	...	"
45	239 - 14	4	5	58	3.3	0.3	89	87	27	...	51	0.7	0.3	...	"
46	179 - 46	5	-2	95	3.3	0.7	102	104	46	...	29	0.3	0.3	...	M 72
47	193 + 34	1	74	115	...	...	92	75	17	...	23	15.4	6.9	...	"
48	290 + 18	2	11	40	1.7	0.8	126	109	74	...	61	3.3	1.4	...	"
49	251 + 43	1	48	80	3.2	0.5	115	95	27	...	37	23.0	9.0	...	"
50	268 + 77	5	64	86	0.6	0.3	115	58	63	29	34	6.0	1.0	9	"
51	23 + 53	5	37	40	-2	1.4	135	80	91	53	61	5.1	2.1	...	M 72
52	310 - 12	5	26	90	3.0	0.2	100	90	23	29	32	2.9	1.4	...	"
53	340 - 21	4	8	66	2.5	0.1	92	86	42	55	46	0.6	0.0	...	"
54	70 + 54	4	25	40	1.1	0.1	113	98	34	51	61	3.3	1.5	...	"
55	74 + 67	5	38	52	0.9	0.1	122	82	63	69	54	5.8	5.5	...	"
56	78 + 59	4	31	47	1.1	1.1	139	105	65	81	57	1.9	12.0	...	"
57	54 + 23	1	17	8	0.2	0.6	92	74	60	120	72	3.7	2.5	...	"
58	48 + 52	5	45	34	2.8	...	110	88	30	45	64	12.8	0.9	...	"
59	47 + 55	4	35	37	1.8	...	124	98	46	60	63	12.7	3.1	...	"
60	48 + 57	5	36	39	0.0	0.7	116	83	55	...	62	6.1	2.8	...	M 72

TABLE IVA—continued

No.	○	Date (G.M.A.T.)	No.	○	Date (G.M.A.T.)	No.	○	Date (G.M.A.T.)
	[°] 34.8	d h m		[°] 138.4	d h m		[°] 128.4	d h m
31	34.8	1897 Apr. 23 11 21	41	138.4	1897 Aug. 9 14 18	51	128.4	1898 July 30 10 43
32	34.8	Apr. 23 11 37	42	140.2	Aug. 11 11 15	52	128.4	July 30 10 53
33	46.5	May 5 12 31	43	16.9	1898 Apr. 5 10 15	53	128.5	July 30 11 10
34	46.5	May 5 12 49	44	28.6	Apr. 17 09 40	54	139.1	Aug. 10 11 17
35	130.6	Aug. 1 12 14	45	28.6	Apr. 17 09 54	55	139.1	Aug. 10 11 56
36	131.6	1897 Aug. 2 10 46	46	28.7	1898 Apr. 17 10 28	56	139.1	1898 Aug. 10 12 12
37	131.7	Aug. 2 13 57	47	28.7	Apr. 17 11 12	57	139.1	Aug. 10 12 22
38	137.3	Aug. 8 09 15	48	28.7	Apr. 17 11 28	58	139.1	Aug. 10 12 47
39	138.4	Aug. 9 13 27	49	33.5	Apr. 22 10 31	59	140.0	Aug. 11 10 56
40	138.4	Aug. 9 13 52	50	115.1	July 16 11 42	60	140.0	Aug. 11 11 11

TABLE IV—continued

No.	Cor- rected Radiant	Wt.	α	ϵ	M	ΔM	H_1	H_2	P	S	V	ΔH_1	ΔH_2	ΔT	Ref.
61	6-22	4	5	58	0.9	0.7	84	79	53	...	51	1.4	0.6	...	M 72
62	49+74	1	48	56	1.2	0.3	91	64	35	54	52	4.1	1.7	...	"
63	51+63	5	45	45	0.7	0.5	115	89	37	...	59	7.3	4.4	...	"
64	53+56	3	40	38	0.9	0.2	103	82	31	61	62	4.2	3.7	...	"
65	48+58	5	45	40	-2.6	1.0	124	87	53	63	61	5.7	2.2	...	"
66	42+57	4	49	40	0.7	1.0	118	92	35	...	61	5.1	1.0	...	M 72
67	60+62	5	44	45	2.1	0.0	118	96	30	...	59	0.9	0.6	...	"
68	74+52	5	28	37	0.9	0.0	112	91	43	...	63	3.4	0.0	...	"
69	5+12	1	20	52	-6	...	99	48	147	35	54	5.0	4.4	...	"
70	66+34	4	6	19	-2	0.5	110	98	87	66	70	2.0	1.6	27	"
71	350+5	5	44	93	0.9	0.9	131	86	64	...	30	22.5	8.7	...	M 72
72	307+42	5	74	105	3.4	0.2	91	81	10	12	26	8.7	3.8	...	...
73	343+0	3	39	102	2.4	1.3	97	79	29	...	27	1.7	5.8	...	M 72
74	253+41	1	47	113	-0.4	0.5	168	142	35	...	23	6.5	3.3	...	"
75	133+34	3	69	124	-2.5	...	119	30	96	15	21	31.5	9.0	...	"
76	331+26	5	30	42	1.0	0.9	96	57	77	44	60	6.9	20.7	85	M 72
77	321+43	1	54	61	4.3	0.0	81	72	11	22	49	4.2	2.9	5	"
78	234-24	5	18	136	1.9	0.7	73	66	21	14	19	0.4	2.5	32	"
79	298+60	4	72	79	3.5	1.0	61	58	4	5	38	3.1	0.3	85	"
80	263+33	5	69	109	3.2	0.2	120	103	18	35	24	8.3	4.9	...	"
81	317-34	5	5	88	3.6	0.1	88	86	20	21	33	0.2	0.4	34	M 72
82	329+39	5	64	63	-0.1	0.9	103	80	25	37	48	0.1	5.8	15	"
83	336-13	2	19	68	1.3	0.3	96	80	50	32	44	6.2	1.5	...	"
84	41+57	5	34	40	1.4	0.4	131	104	49	50	61	11.6	3.8	0	"
85	76+59	5	30	47	1.1	0.0	110	91	36	39	57	0.1	3.2	6	B 9
86	47+64	4	42	47	1	...	120	81	58	...	57	12.4	5.7	...	B 8
87	45+58	3	37	40	0.0	0.0	129	86	71	81	61	6.4	9.6	31	M 72
88	45+57	1	37	39	-3	...	106	54	86	69	62	0.8	19.0	...	B 8
89	53+47	5	19	28	0.0	0.6	108	95	38	36	67	8.9	7.3	...	M 72
90	57+38	5	37	20	-5	...	126	96	49	37	70	14.5	1.6	...	"

TABLE IVA—continued

No.	○	Date (G.M.A.T.)	No.	○	Date (G.M.A.T.)	No.	○	Date (G.M.A.T.)
61	140.0	1898 Aug. 11 11 51	71	174.8	1898 Sept. 16 10 55	81	127.2	1899 July 29 11 09
62	140.0	Aug. 11 11 57	72	175.7	Sept. 17 10 04	82	127.2	July 29 11 13
63	140.0	Aug. 11 12 17	73	175.8	Sept. 17 11 52	83	130.1	Aug. 1 11 10
64	140.0	Aug. 11 12 20	74	180.6	Sept. 22 09 13	84	134.9	Aug. 6 10 36
65	140.0	Aug. 11 12 24	75	341.8	1899 Mar. 1 08 51	85	137.8	Aug. 9 11 58
66	140.0	1898 Aug. 11 12 40	76	98.6	1899 June 29 10 33	86	138.8	1899 Aug. 10 11 31
67	140.0	Aug. 11 12 46	77	107.2	July 8 10 42	87	139.7	Aug. 11 11 03
68	145.8	Aug. 17 11 42	78	107.2	July 8 10 55	88	139.7	Aug. 11 11 10
69	149.5	Aug. 21 09 16	79	107.2	July 8 10 59	89	142.5	Aug. 14 09 31
70	173.7	Sept. 15 08 17	80	114.8	July 16 11 10	90	160.9	Sept. 2 12 04

TABLE IV—continued

No.	Cor- rected Radiant	Wt.	<i>a</i>	<i>ε</i>	<i>M</i>	ΔM	<i>H</i> ₁	<i>H</i> ₂	<i>P</i>	<i>S</i>	<i>V</i>	ΔH_1	ΔH_2	ΔT	Ref.
			[°]	[°]										%	
91	139 + 64	3	30	47	2.2	0.2	81	71	21	24	57	1.2	1.1	...	M 72
92	34 - 12	1	24	83	2.4	0.8	78	72	15	23	35	3.0	2.4	17	"
93	39 + 15	5	49	98	2.5	0.7	91	77	18	30	28	2.4	1.1	...	"
94	73 + 33	5	72	64	3.0	0.5	137	72	69	62	47	4.2	18.8	...	"
95	126 - 6	1	30	28	2.0	0.1	127	81	91	139	67	13.9	3.9	7	B 9
96	230 + 50	1	22	64	1.0	0.2	79	57	57	29	47	10.5	1.1	31	M 72
97	97 - 15	2	27	113	0.9	0.3	93	78	35	21	23	1.6	2.7	36	"
98	184 + 39	5	55	108	-6	...	98	56	50	...	25	3.0	17.6	...	"
99	263 + 16	5	7	42	1.9	0.1	88	85	18	...	60	0.4	0.7	...	M 72
100	182 + 43	1	77	110	1.9	0.1	82	69	13	13	24	1.6	0.8	12	"
101	257 + 45	3	43	77	3.8	2.0	68	59	13	43	39	...	...	...	B 9
102	168 + 1	1	43	132	2.9	0.6	145	97	72	22	19	12.4	5.3	...	M 72
103	327 + 6	1	13	26	2.8	1.3	103	58	186	39	68	0.9	4.8	7	"
104	296 - 20	5	14	89	-5	1.0	76	44	135	35	32	2.7	1.8	1	"
105	18 + 49	4	39	39	-2.1	1.2	128	91	57	51	62	7.9	1.6	33	"
106	355 + 50	2	50	49	1.3	0.3	80	57	30	33	56	5.1	5.7	...	...
107	6 + 46	1	44	41	-4.1	0.3	110	65	65	84	61	14.4	0.2	8	...
108	123 + 72	5	37	58	3.2	0.6	114	95	31	65	51	8.2	2.0	...	...
109	348 - 10	4	24	135	-5	2.0	99	75	62	45	19	3.8	1.2	29	M 72
110	62 + 12	2	37	81	-4	1.4	98	66	55	19	37	0.4	7.3	...	...
111	109 + 22	2	41	65	-0.6	0.4	98	73	39	...	46	5.9	3.2	...	M 72
112	53 + 53	3	32	34	-4	0.4	129	100	56	62	64	17.4	1.2	...	"
113	152 + 24	4	46	12	-1.5	1.0	129	76	74	126	72	2.3	11.2	10	"
114	282 + 23	4	60	103	2.3	0.8	103	80	26	27	26	10.6	5.6	...	"
115	353 + 0	1	22	41	-2.8	0.1	120	91	79	49	61	2.8	0.4	...	"
116	352 + 64	1	63	58	0.7	0.6	88	81	9	...	51	7.4	1.1	...	B 11
117	296 - 10	4	31	101	-0.5	1.3	86	76	18	20	27	1.8	0.2	...	M 72
118	191 + 23	2	19	127	0.9	0.3	99	84	45	14	20	1.5	4.0	...	"
119	304 + 54	5	83	83	0.8	...	95	82	14	70	35	9.5	5.0	9	"
120	61 + 56	5	31	39	-3	0.9	125	102	43	61	62	7.9	2.6	...	B 11

TABLE IVA—continued

No.	°	Date (G.M.A.T.)	No.	°	Date (G.M.A.T.)	No.	°	Date (G.M.A.T.)
	[°]	d h m		[°]	d h m		[°]	d h m
91	193.2	1899 Oct. 5 10 55	101	31.1	1900 Apr. 20 10 18	111	262.2	1900 Dec. 13 10 19
92	198.1	Oct. 10 11 06	102	32.1	Apr. 21 10 31	112	143.1	1901 Aug. 15 10 48
93	229.2	Nov. 10 13 16	103	43.8	May 3 13 58	113	232.7	Nov. 14 15 23
94	229.2	Nov. 10 13 37	104	113.6	July 15 10 13	114	124.6	1902 July 27 11 28
95	232.3	Nov. 13 15 13	105	117.5	July 19 11 49	115	124.6	July 27 11 37
96	282.9	1900 Jan. 2 11 58	106	127.9	1900 July 30 10 42	116	124.6	1902 July 27 11 57
97	308.3	Jan. 27 11 10	107	128.9	July 31 10 52	117	126.5	July 29 11 27
98	8.4	Mar. 28 08 30	108	160.7	Sept. 2 11 58	118	138.0	Aug. 10 10 08
99	8.5	Mar. 28 10 34	109	208.9	Oct. 21 11 59	119	138.0	Aug. 10 10 26
100	8.5	Mar. 28 10 44	110	232.0	Nov. 13 15 48	120	140.0	Aug. 12 11 06

TABLE IV—continued

No.	Cor- rected Radiant	Wt.	<i>a</i>	<i>ε</i>	<i>M</i>	ΔM	<i>H</i> ₁	<i>H</i> ₂	<i>P</i>	<i>S</i>	<i>V</i>	ΔH_1	ΔH_2	ΔT	Ref.
														%	
121	193-10	4	31	103	-3	1.0	90	45	87	26	26	1.6	4.3	15	M 72
122	144+40	5	68	154	0.3	0.5	72	43	31	10	17	6.6	3.0	29	"
123	270+32	3	48	61	-3	2.0	127	72	74	59	49	3.2	2.6	25	"
124	34+3	2	1	20	-2.5	...	107	105	76	45	70	0.9	0.2	...	"
125	57+21	5	48	85	-7	1.0	108	60	64	49	35	2.6	1.0	19	"
126	61+24	5	59	80	-7	0.7	123	56	79	...	37	10.1	16.3	...	M 72
127	150+27	5	36	14	-8	1.8	129	78	85	...	71	35.1	8.9	...	"
128	152+25	3	57	13	-4.3	0.7	132	90	50	86	71	19.6	11.6	24	"
129	153+25	5	60	13	-1.3	0.7	110	70	47	60	71	17.0	16.8	29	"
130	160+11	3	44	13	-3.8	0.7	115	93	31	100	71	10.3	5.2	...	"
131	51+19	5	55	93	v.b.	...	96	59	46	49	30	17.2	2.5	26	M 72
132	4+43	1	70	45	-5	...	79	51	30	20	59	2.0	0.9	...	"
133	145+26	1	26	13	-4	0.9	175	109	146	134	71	7.7	3.4	27	"
134	207+63	5	35	74	-2.2	0.3	90	50	70	34	41	5.4	5.6	...	M 76
135	57+20	4	60	101	-2.5	1.5	110	68	49	26	27	18.9	17.1	46	"
136	271+33	5	30	62	0.4	0.0	109	71	75	...	48	9.9	17.5	...	M 76
137	268-7	5	34	97	2.8	0.5	95	82	22	...	29	7.8	7.8	...	"
138	27+62	5	50	47	0.6	...	133	70	81	...	57	14.3	2.9	...	"
139	48-6	5	20	78	...	...	79	72	21	...	38	2.3	5.2	...	"
140	37-16	2	11	96	1	...	98	77	107	...	29	2.6	0.1	...	"
141	53-4	4	17	77	...	...	89	72	56	...	39	1.2	2.7	...	M 76
142	235+30	5	14	91	-3	...	97	80	67	...	31	8.0	4.1	...	"
143	139+17	1	31	91	-4	...	71	35	69	35	31	8.5	3.9	...	"
144	149+12	3	46	87	...	...	149	87	86	...	33	25.7	1.5	...	"
145	117+13	5	51	122	...	...	126	93	42	...	21	6.3	13.5	...	M 76
146	105+50	2	72	152	...	...	140	125	15	...	17	4.1	2.1	...	M 76
147	70+16	4	15	132	...	...	100	69	111	23	19	11.4	1.8	...	"
148	148-7	5	35	118	2.5	0.8	79	62	28	62	22	1.7	7.3	...	J 27
149	52+67	4	33	109	-5	0.9	83	56	49	21	24	13.6	4.3	44	"
150	109+35	5	65	67	-5.5	2.0	162	99	69	93	45	5.7	28.9	...	"

TABLE IVA—continued

No.	○	Date (G.M.A.T.)	No.	○	Date (G.M.A.T.)	No.	○	Date (G.M.A.T.)
		d h m			d h m			d h m
121	29.3	1903 Apr. 19 10 07	131	236.2	1903 Nov. 18 12 44	141	214.4	1913 Oct. 27 09 11
122	32.3	Apr. 22 10 23	132	139.5	1904 Aug. 11 13 06	142	219.4	Nov. 1 08 26
123	32.3	Apr. 22 12 33	133	233.9	Nov. 15 12 38	143	316.7	1914 Feb. 5 08 00
124	139.7	Aug. 12 10 55	134	195.7	1912 Oct. 8 08 32	144	324.9	Feb. 13 10 18
125	233.1	Nov. 15 10 19	135	249.9	Dec. 1 10 37	145	329.9	Feb. 18 08 20
126	233.2	1903 Nov. 15 13 42	136	32.7	1913 Apr. 22 10 22	146	1.9	1914 Mar. 22 09 08
127	233.2	Nov. 15 13 59	137	96.2	June 27 11 36	147	29.5	Apr. 19 09 45
128	233.3	Nov. 15 16 45	138	136.3	Aug. 8 11 42	148	3.2	1917 Mar. 23 10 47
129	233.3	Nov. 15 17 14	139	209.4	Oct. 22 09 40	149	41.5	May 1 14 00
130	233.3	Nov. 15 17 22	140	214.4	Oct. 27 08 23	150	261.4	1919 Dec. 13 11 53

TABLE IV—continued

No.	Cor- rected Radiant	Wt.	<i>a</i>	<i>ε</i>	<i>M</i>	ΔM	<i>H</i> ₁	<i>H</i> ₂	<i>P</i>	<i>S</i>	<i>V</i>	ΔH_1	ΔH_2	ΔT	Ref.
151	218°+20°	5	56°	103°	0.3	0.6	93	75	23	49	26	6.4	0.2	16	
152	308+10	5	27	28	-1.5	1.3	119	101	38	68	67	3.1	1.6	8	
153	51+55	5	47	51	-0.1	0.6	154	112	58	56	55	30.6	5.9	54	
154	108+4	5	24	32	-5	...	112	87	65	88	65	14.0	3.4	...	J 32
155	211+43	5	20	51	0.8	...	119	105	37	60	55	7.9	7.9	...	B 14
156	228+32	4	71	109	-5	0.2	105	64	44	14	24	3.1	3.1	...	B 14
157	45+43	3	7	84	2.4	0.3	121	103	126	40	35	1.2	0.3	...	"
158	289-6	5	33	97	0.9	0.2	104	90	27	11	29	6.1	14.3	58	"
159	48+50	5	25	44	1.2	0.2	109	101	19	50	59	2.4	1.4	12	"
160	20+45	5	34	35	0.3	0.4	75	66	16	35	64	4.4	2.7	48	"
161	36+43	5	35	30	-2.4	0.6	135	119	28	45	66	11.3	7.0	64	B 14
162	38+45	4	48	31	-3.4	1.8	104	84	27	60	66	4.0	5.8	...	
163	307-8	1	32	91	-0.5	...	51	45	13	...	31	4.5	1.9	...	
164	326+31	5	68	66	1.3	0.2	96	85	12	20	46	6.8	5.4	3	B 14
165	179+58	2	32	92	2.1	0.4	89	77	23	10	31	14.1	10.0	53	"
166	50+74	1	49	55	-0.1	...	113	106	9	18	52	...	...	...	B 14
167	35+2	1	35	82	0.3	0.4	128	78	89	50	36	10.2	10.9	43	"
168	139+39	5	37	24	2.0	0.2	123	103	34	36	68	9.8	0.4	39	
169	126+43	5	37	34	-3.4	1.1	105	80	41	34	64	9.6	14.9	11	
170	312+73	5	37	101	0.2	0.1	95	84	19	12	27	...	0.4	...	O 46
171	313+54	5	40	75	0.7	0.1	123	105	28	29	40	2.8	11.4	63	
172	346-16	4	3	60	0.6	0.3	107	99	124	74	49	0.1	0.6	...	
173	26+59	1	56	44	v.b.	...	102	52	61	59	59	21.1	5.2	45	
174	42+58	5	43	41	-6	...	108	85	33	55	61	0.7	1.0	8	
175	75+58	2	27	45	-4	1.3	114	102	26	49	59	11.0	9.6	57	
176	48+53	5	43	35	-4.8	1.0	115	92	34	48	64	7.4	5.8	7	
177	42+58	5	35	40	-6	...	87	72	26	42	61	5.2	3.6	...	
178	50+58	5	39	40	-1.4	0.0	123	97	41	63	61	1.4	4.9	5	
179	185+60	5	23	62	-0.5	...	136	93	105	40	48	24.3	2.2	...	O 46
180	57+9	4	5	72	-0.5	...	76	61	131	13	42	2.5	1.9	...	

TABLE IVA—continued

No.	☉	Date (G.M.A.T.)	No.	☉	Date (G.M.A.T.)	No.	☉	Date (G.M.A.T.)
151	44.4	1921 May 4 12 59	161	125.5	1922 July 28 12 09	171	30.3	1923 Apr. 20 13 14
152	44.4	May 4 13 45	162	127.5	July 30 13 36	172	130.0	Aug. 2 10 08
153	192.5	Oct. 5 09 17	163	128.3	July 31 11 17	173	136.8	Aug. 9 12 32
154	224.5	Nov. 6 12 26	164	128.4	July 31 12 45	174	137.7	Aug. 10 11 41
155	283.2	1922 Jan. 3 11 19	165	142.7	Aug. 15 10 30	175	138.6	Aug. 11 11 07
156	62.4	1922 May 23 10 34	166	142.7	1922 Aug. 15 11 40	176	138.7	1923 Aug. 11 12 24
157	62.4	May 23 10 42	167	204.2	Oct. 17 10 30	177	139.5	Aug. 12 10 11
158	116.9	July 19 10 43	168	207.3	Oct. 20 14 04	178	140.6	Aug. 13 11 32
159	116.9	July 19 11 32	169	233.2	Nov. 15 11 10	179	200.9	Oct. 14 09 36
160	117.8	July 20 11 21	170	356.7	1923 Mar. 17 10 08	180	217.7	Oct. 31 06 49

TABLE IV—continued

No.	Cor- rected Radiant	Wt.	<i>a</i>	ϵ	<i>M</i>	ΔM	<i>H</i> ₁	<i>H</i> ₂	<i>P</i>	<i>S</i>	<i>V</i>	ΔH_1	ΔH_2	ΔT	Ref.
			^o	^o										%	
211	237 - 18	2	19	62	-3	...	93	75	56	58	48	1.5	1.0	...	O 52
212	158 + 36	5	15	37	-1.9	0.4	126	106	74	40	63	8.3	9.1	4	"
213	88 + 13	4	37	30	-0.5	0.1	131	104	44	65	66	2.7	4.3	56	"
214	144 + 61	5	45	44	0.7	0.3	102	90	18	41	59	5.3	6.5	35	
215	130 + 60	5	35	40	1.0	1.6	116	101	25	28	61	10.8	8.5	2	O 52
216	93 + 15	5	27	27	-0.5	0.9	117	102	33	34	67	2.5	1.9	48	O 52
217	93 + 18	5	28	26	0.9	0.7	112	101	23	33	68	7.2	9.5	29	
218	101 + 16	4	31	19	-1.6	0.5	107	96	22	32	70	2.9	0.2	...	O 52
219	105 + 4	3	20	23	1.6	0.6	112	106	17	25	69	1.4	0.8	24	
220	349 - 1	5	26	128	0.8	0.6	85	78	15	23	20	0.5	2.3	24	
221	87 + 36	5	67	25	-0.3	0.6	110	88	23	25	68	3.7	6.2	73	
222	110 + 13	5	38	9	2.1	0.0	114	105	15	21	72	3.8	3.9	...	O 52
223	98 + 19	5	50	22	-0.3	0.2	112	98	19	41	69	5.1	0.3	47	
244	105 + 25	5	53	15	2.4	0.7	97	82	19	17	71	1.8	3.7	...	
225	156 + 43	4	43	37	0.4	0.8	120	99	31	81	63	7.1	5.1	15	
226	96 + 14	5	53	25	-1.0	0.5	101	74	34	68	68	...	3.7	...	
227	102 + 5	3	42	24	0.5	0.9	129	103	39	139	68	11.6	22.4	...	
228	94 - 26	4	13	53	0.5	0.1	123	115	37	42	54	1.5	1.7	45	

TABLE IVA—continued

No.	⊙	Date (G.M.A.T.)	No.	⊙	Date (G.M.A.T.)
	^o	d h m		^o	d h m
211	32.0	1928 Apr. 21 12 12	221	207.8	1928 Oct. 20 14 20
212	205.7	Oct. 18 12 50	222	207.8	Oct. 20 14 20
213	205.7	Oct. 18 12 57	223	207.8	Oct. 20 14 36
214	205.8	Oct. 18 13 58	224	207.8	Oct. 20 14 47
215	207.6	Oct. 20 11 15	225	207.8	Oct. 20 15 28
216	207.6	1928 Oct. 20 11 48	226	207.8	1928 Oct. 20 16 10
217	207.6	Oct. 20 11 51	227	207.8	Oct. 20 16 16
218	207.7	Oct. 20 12 47	228	207.8	Oct. 20 16 25
219	207.7	Oct. 20 12 48			
220	207.7	Oct. 20 12 49			

PHOTOVISUAL MAGNITUDES OF SOUTHERN STARS

A. W. J. Cousins

(Received 1943 April 6)

Summary

Photovisual magnitudes of rather more than a hundred stars in the southern hemisphere have been determined using Fabry's method. The scale and zero point have been standardised against the brighter stars of the Pleiades using Calder's magnitudes. There are differences of scale and zero point and a large colour coefficient when compared with visual magnitudes reduced to the Harvard Photovisual System. The mean error of a magnitude is believed to be of the order of $+0^m.02$.

1. *Introduction.*—The successful use of Fabry's method by the late E. G. Williams for the determination of the colours and magnitudes of southern B-type stars * has led to another series using the same method, at the Durban Observatory. The main object of this series has been to determine more accurate photovisual magnitudes for naked eye stars in the southern hemisphere. No attempt has been made to determine the colours, but stars of all spectral types are included. The original programme included stars from the second to the fifth magnitude, and comparisons with the Pleiades were made to determine the scale and zero point. At a later stage the programme was extended to include brighter and fainter stars.

War conditions have imposed severe limitations on the work, and it has not been possible to make the series as complete as could be wished. It is hoped that it will be possible to resume the observations at some future date, but it seems opportune to present the results already obtained without further delay.

2. *Equipment.*—The camera, which is a self-contained unit, was designed to use roll film, as suitable plates were practically unobtainable. It is provided with a lens of 0.25 inch focal length, and two apertures 1 mm. wide by 31 mm. long for calibrating the film. Both the lens and the apertures are normally covered by shutters. In front of the lens shutter is a diaphragm with an aperture 2.5 mm. in diameter, which can be reduced to 1.6 mm. when there is danger from sky fog. When using the latter more care is required for setting and guiding, but there is no evidence of any systematic effect. The film winding arrangements follow orthodox lines, but the ruby window is elongated and graduated to enable 20 star images and two pairs of calibration strips to be exposed to one standard "frame." Behind the lens is a diaphragm with an aperture 5 mm. in diameter, the rear face of which lies in the focal plane of the lens. The film is held against this during exposure by a spring loaded disk.

When in use, the camera was attached to the draw tube of the 8-inch Grubb refractor or its 3-inch finder, with the lens diaphragm at the focus and a colour filter four or five centimetres in front. The finder was used for the brighter stars.

Kodak "Super XX" and "Panatomic X" films were used, but principally the former, in conjunction with either a Wratten 11 or a Zeiss Ikon 2 colour filter. The former filter was chosen to obtain a close approximation to the colour sensitivity of the eye, but, as the absorption is about a magnitude greater than that of the Zeiss filter, the latter

* The late E. G. Williams and H. Knox-Shaw, *M.N.*, 102, 226, 1942.

possesses obvious advantages for the fainter stars. All the observations were reduced to the system of the Wratten filter and Super XX film, the following colour correction coefficients being adopted:—

Super XX film;	Wratten filter	0.00 × C.I.
„ „	Zeiss filter	+0.04
Panatomic X film;	Wratten filter	-0.07
„ „	Zeiss filter	+0.03

No measurement of the effective wave-length has been made. From manufacturer's data a value near 5200 Å. is expected, but a comparison with Williams' Red and Green magnitudes * favours 5400 Å.

With one exception, the films were tank developed. It was not possible to control the conditions, and the quality of the films varies somewhat. In most cases the Eberhard effect is rather pronounced. The remaining film was brush developed by Dr. Redman at the Radcliffe Observatory, and the advantage of this procedure is evident.

3. *Calibration*.—At the suggestion of Dr. Redman a sensitometer was designed to give a continuously graded image. It consists of a box 5 inches square by 38 inches long, at one end of which is a logarithmically shaped aperture cut in a brass plate.† At an intermediate point is a diaphragm with a slit 1 mm. wide, and at the other end an opening into which the camera fits. In front of the aperture and inclined at 45° to it is a porcelain plate illuminated normally from the front by an internally frosted electric lamp. The box is well diaphragmed between the aperture and slit, and this is so placed as to give a 2.5 : 1 reduction of the scale of the logarithmic aperture and produce a gradation of one magnitude per centimetre at the surface of the film. The total range is over 3 magnitudes, but not more than 2.5 can be used in practice. Tests showed a slight deficiency of illumination towards the ends of the strips, and this was taken care of in the reductions. No filters were used.

To test the scale value of the sensitometer, thirteen sets of observations were made of the brighter stars of the Pleiades. The observed magnitude differences have been compared with Calder's magnitudes from *Harvard Annals*, 105, 453, 1937, with the following results (Cousins - Calder):—

	$\frac{\Delta m}{m}$	m.e.
Super XX film (tank developed)	-0.018	±0.018
Panatomic X film (brush developed)	+0.003	±0.012

The scale error is evidently small and has been neglected.

The mean magnitudes obtained from these observations are given in Table I, together with Calder's values. The observations of Pleione are discordant, but this star is known to be variable and was not used in this investigation.

TABLE I

Star	Magnitude		Star	Magnitude	
	Cousins	Calder		Cousins	Calder
Alcyone	2.97	2.97	Taygeta	4.41	4.37
Atlas	3.71	3.72	Pleione	5.33	5.14
Electra	3.77	3.77	Celæno	5.57	5.59
Maia	3.96	3.98	Asterope	5.93	5.91
Merope	4.27	4.30			

* *Loc. cit.*

† A somewhat similar apparatus was used by E. S. King for making photographic wedges. See *H.A.*, 41, 237, 1902.

It was not always possible to give the star images and calibration strips the same exposure and this necessitated corrections to the observed magnitude differences, the following factors being used:—

$$\text{Super XX film} \quad 1 - 0.06 \log \frac{t_s}{t_c}$$

$$\text{Panatomic X film} \quad 1 - 0.03 \log \frac{t_s}{t_c}$$

The star exposures (t_s) were of 30, 60 or 120 seconds duration. The calibration exposures (t_c) ranged between $7\frac{1}{2}$ and 120 seconds.

4. *Measurement and Reduction.*—Rather more than one quarter of the images were measured with a photo-electric cell and amplifier on the measuring machine at the Radcliffe Observatory, the great majority by Dr. Knox-Shaw. The remainder were measured visually by the writer by placing two films in contact and using the calibration strips as photographic wedges. Visual measures were repeated several times to reduce the measuring error. In either case magnitude differences could have been obtained directly from linear measurements along the strips, but it was found better when reducing the photo-electric measures to follow the recognised procedure of determining the plate characteristic from the calibration strips and using this for reducing the measures. There is no doubt that the photo-electric method is superior to visual measuring.

As the stars are widely distributed over the sky, it was necessary to correct the magnitudes for atmospheric extinction. Earlier visual observations had shown large variations from day to day—at least from $0^m.20$ to $0^m.30$ (at zenith)—and the photovisual observations confirm this result. An attempt was made to determine the transparency each night by observing stars at low altitude, and this formed a useful basis, but as the work progressed it became apparent that the atmospheric transmission not only varied with time, but might vary appreciably from one part of the sky to another, even on an apparently clear night. The writer believes that this is the most serious source of error in employing Fabry's method for this class of work. Because of this variation in the transparency it was found preferable to calculate the total extinction instead of reducing the magnitudes of the zenith only.

The zero point of the magnitude system was determined from eleven comparisons with the brighter stars of the Pleiades on five nights in 1941 and 1942. Calder's magnitudes were adopted. The comparatively low altitude attained by the Pleiades is a disadvantage, but no more suitable standards are available for bright stars.

The m.e. of the zero point thus determined for stars in Table II is $\pm 0^m.013$. There is no evidence of any change with R.A.; but where secondary comparisons are involved, as for stars in Tables III and IV, the uncertainty is greater.

5. *Results.*—The tables that follow give the mean magnitudes derived from the observations. They contain, in addition, visual magnitudes reduced to the Harvard Photovisual System from *Harvard Mimeograms*, Series III, Nos. 1 and 2.

The weights given in the tables are on the basis of unit weight for a normal image of medium density. Observations subject to greater uncertainty were used with lower weights or rejected. The sum of the weights is rather less than two-thirds of the number of images secured.

The average mean error of an observation of unit weight is $\pm 0^m.036$, but certain series gave values as low as $\pm 0^m.027$. There are variations from star to star, but these appear to be for the most part accidental. The mean error of the final magnitudes is usually about $\pm 0^m.01$ or $\pm 0^m.02$.

6. *Comparisons.*—Comparison with the Harvard magnitudes shows differences of scale and a large colour coefficient. For stars brighter than magnitude 6.0 both coefficients remain approximately constant and the relation may be expressed:

$$\text{Cousins} - \text{H.Mim.} = 0.04(m - 2.7) + 0.23 \times \text{C.I. (HPv)}.$$

TABLE II
Main Programme

H.D.	Name	Spect.	Cousins		H.Mim. Magn.	H.D.	Name	Spect.	Cousins		H.Mim. Magn.
			Magn.	Wt.					Magn.	Wt.	
23817	β Reti	Ko	4.09	6	3.64	80230	g Cari	K5	4.50	27	3.96
27256	α Reti	G5	3.54	5	3.26	80404	i Cari	Fo	2.33	5	2.18
27290	γ Dora	F5	4.46	10	4.28	81101	k Cari	Ko	4.97	24	4.70
29305	α Dora	Aop	3.37	9	3.39	81188	κ Veln	B3	2.51	5	2.51
33262	ζ Dora	F8	4.95	6	4.68	82668	N Velr	K5	3.29	14	2.82
39014	δ Dora	A5	4.49	17	4.48	83183	h Cari	B5	4.18	15	4.08
39060	β Pict	A3	3.98	19	3.89	83944	m Cari	B9	4.61	31	4.57
50241	α Pict	A5	3.37	11	3.25	85123/4	v Cari	Fo	3.03	9	3.01
53705/6	...	Go + Go	5.40	3	5.36	86440	ϕ Velr	B5	3.59	7	3.58
55526	...	K2	5.30	5	4.94	89080	ω Cari	B8	3.39	12	3.44
55864/5	γ Voln	Ko + Go	3.74	7	3.56	89388	q Cari	K5	3.45	15	3.23
55892	I Pupp	Fo	4.62	8	4.39	91465	p Cari	B5p	3.33	12	3.44
56022	L^1 Pupp	Aop	4.98	8	4.98	93030	θ Cari	Bo	2.77	6	2.91
56455	...	Aop	5.89	2	5.74	94510	u Cari	Ko	3.88	14	3.78
56456	...	B8	4.87	5	4.78	160635	η Pavo	Ko	3.69	3	3.42
57150	v^1 Pupp	B3	4.91	11	4.67	165040	π Pavo	A5	4.43	15	4.39
57219	v^2 Pupp	B3	5.23	10	5.14	168339	ξ Pavo	K2	4.51	12	4.08
57623	δ Voln	F5	4.14	5	3.94	169978	v Pavo	B8	4.73	13	4.70
59717	σ Pupp	K5	3.32	4	2.98	171759	ζ Pavo	Ko	4.11	11	3.94
62226	...	B2	5.52	4	5.45	172555	...	A2	4.93	13	4.85
63922	P Pupp	Bo	4.16	10	4.17	173948	λ Pavo	B2	3.91	11	4.28
64440	α Pupp	G5	3.85	7	3.62	175329	ω Pavo	Ko	5.38	7	4.98
64740	...	B3	4.69	14	4.71	175986	...	Go	6.19	6	5.84
64760	γ Pupp	B1	4.35	10	4.20	177389	...	Ko	5.56	8	5.14
64802	...	B3	5.60	4	5.40	188228	ϵ Pavo	Ao	4.02	5	4.02
66005/6	...	B3 + B3	5.71	4	5.67	197635	σ Pavo	Ko	5.72	5	5.31
76728	c Cari	B8	3.96	13	3.88	201371	o Pavo	Ma	5.34	6	4.84
78045	α Voln	A5	4.13	14	4.12	203881	...	Mb	6.12	5	5.26
79351	α Cari	B3	3.50	13	3.43	207241	o Indi	K2	5.85	5	5.33
79447	i Cari	B3	4.05	29	4.06						

TABLE III

Bright Stars

H.D.	Name	Spect.	Cousins		H.Mim. Magn.	H.D.	Name	Spect.	Cousins		H.Mim. Magn.
			Magn.	Wt.					Magn.	Wt.	
71129/30	ϵ Cari	Ko + B	1.95	4	1.58	148478/9	α Scor	Ma + A3	1.19	2	1.05
74956	δ Velr	Ao	2.13	4	1.93	150798	α Trau	K2	1.98	4	1.72
80007	β Cari	Ao	1.65	3	1.72	158427	α Arae	B3p	3.04	2	2.82
108248/9	α Cruc	B1 + B1	0.80	2	0.90	158926	λ Scor	B2	1.66	3	1.66
108903	γ Cruc	Mb	1.76	4	1.46	159532	θ Scor	Fo	1.90	3	1.97
110304	γ Cent	Ao	2.29	2	2.30	169022	ϵ Sgtr	Ao	1.81	2	1.86
111123	β Cruc	B1	1.30	3	1.35	193924	α Pavo	B3	2.07	3	1.97
122451	β Cent	B1	0.75	1	0.72	209952	α Grus	B5	1.89	1	2.10

REMARKS

Abbreviations: m.e. = mean error of a single observation.

CPv = present series.

Cv = visual observations by writer.

WPv = E. G. Williams' series adjusted to CPv system.

Hc = H.Mim. adjusted to CPv system.

KPv = E. S. King's photovisual observations from *H.A.*, 85, 185, 1930.

H.D.

- 53705/6 Double, m.e. $\pm 0^m.06$.
 57623 m.e. $\pm 0^m.05$.
 66005/6 Double, m.e. $\pm 0^m.05$.
 80404 m.e. $\pm 0^m.06$.
 82668 *N* Velr. Suspected variable, see *Trans. I.A.U.*, 6, 239, 1939. Observed 9 nights: 1941 December to 1942 July; m.e. $\pm 0^m.04$. CPv = 3.29, Cv = 3.26, Hc = 3.15. No evidence of variability.
 173948 λ Pavo. Observed 12 nights: 1942 May to August; m.e. $\pm 0^m.05$. CPv (1942) = 3.91, WPv (1940) = 4.00, Cv (1937-8) = 4.13, Hc = 4.32. Variability suspected. See F. Möller, *A.N.*, 165, 377, 1904, and Williams and Knox-Shaw, *loc. cit.*
 203881 m.e. $\pm 0^m.03$. CPv = 6.12, Hc = 5.76.
 158427 CPv = 3.04, WPv = 2.93, KPv = 2.81, Hc = 2.80.
 110432 In "Coal Sack." CPv = 5.42, WPv = 5.45, Hc = 5.95. Variability suspected. See Williams and Knox-Shaw, *loc. cit.*

TABLE IV

Faint Stars

H.D.	Spect.	Cousins		H.Mim. Magn.	H.D.	Spect.	Cousins		H.Mim. Magn.
		Magn.	Wt.				Magn.	Wt.	
A. Carina									
92207	A2p	5.65	1	5.51	94144	B5	7.30	4	6.92
92740	Ocp	6.67	2	6.36	94173	Ao	7.19	4	6.96
93502	Ao	6.62	6	6.39	94493	B1	7.87	2	7.32
93695	B5	6.81	2	6.57	94683	K5	6.21	7	5.84
93737	A2p	6.29	2	6.06	95324	B9	6.47	7	6.21
93943	Ao	6.17	5	6.02	95880	B8	7.35	2	6.97
B. Musca									
102839	G5	5.17	4	4.78	106676	Ao	6.47	5	6.20
104015	B5	7.39	2	6.89	107567	Ko	5.96	8	5.66
104570	Ko	6.73	5	6.20	109867	B2	6.55	2	6.31
104600	B8	6.06	8	6.00	110716	Go	6.43	4	6.24
104878	Ao	5.48	7	5.34	112383	Ao	7.25	2	...
105138	Ko	6.52	5	6.08					
C. Crux									
104933	Ma	6.19	2	5.72	108355	B8	6.23	2	6.09
105437	K2	6.42	2	6.05	108501	Ao	6.29	2	6.20
105841	Fo	6.34	2	6.17	108773	Fo	7.01	3	6.76
106000	A2	6.92	2	6.64	109000	A5	6.13	2	6.08
106068	B2	6.14	2	6.02	109492	G5	6.49	1	6.17
106343	B1	6.47	2	6.27	110432	B1p	5.42	2	5.85
106742	Go	7.35	2	...	110532	Ko	6.70	2	6.30
106943	B9	8.02	2	...	111102	A5	7.38	3	...
107209	B3	7.23	3	6.79	111538	K5	7.51	2	...
107979/80	Go + A2	7.43	1	7.03	111613	A2p	5.96	4	5.88
D. Grus									
211053	Ko	6.47	3	6.00	212180	A2	7.41	2	7.00
212132	Fo	5.94	3	5.76	213658	Ko	7.28	2	6.76

The colour correction is not quite linear and is better defined by the following smoothed values:—

B1	-0.02	Ao	-0.01	Fo	+0.04	Ko	+0.25
B5	-0.01	A5	+0.01	Go	+0.14	K5	+0.32

Two stars give residuals exceeding $0^m.25$, and these were omitted from the above analysis. Both are B-type stars and were observed by Williams, whose magnitudes confirm the present values. There are suspicions of variability.

The mean deviation between the magnitudes of the remaining 78 stars, after applying the above corrections, is $\pm 0^m.09$, rather larger than would be expected if $\pm 0^m.06$ * can be accepted as the average mean error of the Harvard Mimeogram magnitudes used in the investigation.

From a study of stars fainter than magnitude 6.0 there appears to be a marked change of scale below that magnitude, the difference Cousins - H.Mim. increasing rapidly for the fainter stars. The number of red stars available for comparison is too small to say whether the change applies equally to stars of all types.

Through the courtesy of Dr. Knox-Shaw the writer has had access to the magnitudes of those stars in the present programme that were also observed by Williams. These magnitudes were reduced to the International Photovisual System and have the same colour coefficient as the present series, but there is a difference of scale. The relation may be expressed:

$$\text{Cousins} - \text{Williams} = 0.04(m - 4.7).$$

After making this adjustment the mean deviation between the magnitudes of 11 B-type stars included in Table II is $\pm 0^m.025$.

The programme owed its inception largely to the pioneer work and personal encouragement of the late Mr. E. G. Williams, and the writer wishes to express his indebtedness also to Dr. H. Knox-Shaw and Dr. R. O. Redman for much advice and encouragement and for invaluable help in other ways in carrying out the work.

Durban :

1943 January.

* *H.Mim.*, Ser. III, No. 1, 1, 1938.

GALACTIC ABSORPTION AND STAR DISTRIBUTION

George Alter

(Communicated by the Director, Norman Lockyer Observatory)

(Received 1943 April 5)

Summary

A comparison of observed densities with the density function, got by solving the fundamental equation of star counting by means of an assumed luminosity function, is limited by the limits of our knowledge of absolute magnitudes or proper motions of the stars observed. An extension is possible by using colour indices, which can be determined down to the limiting apparent star magnitudes of the photographs. These colour indices and apparent magnitudes (and therefore the derived densities) are, however, affected by selective and total absorption. A relation is here established between the true and the apparent densities in such a case. A transformation factor is deduced, using an analytical expression for the $M-C$ curve, and assuming uniform galactic absorption. This factor, applied to the density function, permits the comparison of observed and computed densities, covering the whole range of observed stars.

1. *Introduction.*—In two earlier papers* (called later GI and GII) a relation was developed between galactic absorption (both total and selective) and star distance. It was also shown that the apparent spatial star distribution is affected differently, by this absorption, for different spectral types if we use observed colour indices (C), instead of spectral types, for distance determinations. This, however, represents a change in the luminosity function and consequently in the fundamental equation of star distribution. This paper derives the changes involved and finally gives a factor for turning the unaffected density value into the affected one; and shows how this may be compared with observed values.

2. *$M-C$ Curve.*—If we want to extend our knowledge of star distribution by the use of colour indices (which are available for nearly all the stars on the photographic plates concerned), instead of absolute magnitudes (which are available only for a comparatively small range), as already urged by Bok†, we have not only to consider the intervening galactic absorption (total and selective), affecting both apparent and absolute magnitudes, but also to find a suitable representation of the relation between absolute magnitude and colour index for convenience in analytical treatment. This is found to be given by the formula

$$M = 12 - e^{-0.667(C-3.65)} \quad (1)$$

which represents very closely the distribution of visual absolute magnitudes of star types between B₃ and M₀ of the main sequence (fig. 1). Within this range, the mean deviation from the observed absolute magnitudes, determined by the Mount Wilson Observatory, the Norman Lockyer Observatory and Lundmark‡, is ± 0.2 . (The following deductions refer always to the visual scale and the suffix v will not be used.)

* *M.N.*, 101, 84; 101, 381, 1941.

† *The Distribution of the Stars in Space*, p. 72, 1937.

‡ *Hdb. d. Ap.*, V, 539, 1932.

The influence of galactic absorption is represented by the absorption modulus

$$b(M, r) = 0.2(a_{pv} - a_s k) \quad (2)$$

(derived in GI) which depends, through k , on absolute magnitude and colour excess (E). If we assume a continuous and uniform absorbing layer then this colour excess is expressed by

$$E = a_s r_0. \quad (3)$$

(The suffixes 0 and 1 indicate, throughout the paper, true and apparent values.)

Formulae (1), (2) and (3) enable us to use the observed, falsified colour indices (or the corresponding falsified absolute magnitudes M_1) as arguments in looking for changes

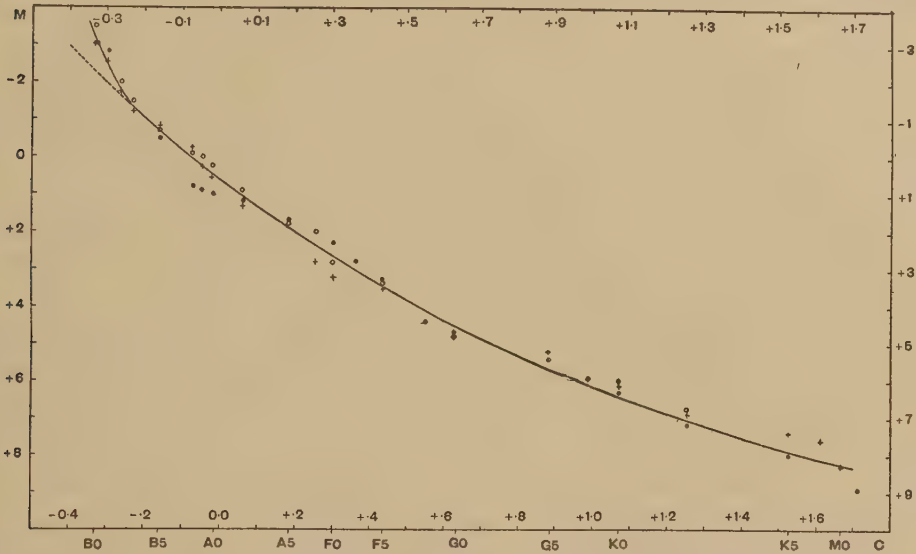


FIG. 1.— M - C curve (dotted line extension of the analytical part).

Crosses: Mount Wilson Observatory Determinations of Absolute Magnitude.

Circles: Norman Lockyer Observatory " " "

Dots: Lundmark " " "

in star distribution introduced by galactic absorption. The following formulae (4) to (9) express the changes of the arguments due to selective absorption.

The differentiation of formula (1) gives

$$\kappa = \frac{dM}{dC} = \frac{2}{3} e^{-\frac{2}{3}(C-3.65)} = \frac{2}{3}(12-M), \quad (4)$$

the limiting value of k for vanishing E ; the value for any E is given by

$$k = \frac{M_1 - M_0}{E} = (12 - M_0) \frac{1 - e^{-\frac{2}{3}E}}{E}. \quad (5)$$

It may be pointed out that, while the change in apparent magnitudes, due to total absorption, is independent of the spectral type, the change in absolute magnitudes, $M_1 - M_0$, due to selective absorption, does depend on the spectral type (or M_0) as will readily be seen from fig. 1 where the same colour excess corresponds to different magnitude changes at different absolute magnitudes. This is also obvious from formula (5).

κ depends only on C (or M) and

$$\frac{d\kappa}{dC} = -\frac{2}{3}\kappa; \quad (6)$$

but, as pointed out in the earlier papers, k depends on C and E , and the relation, which was given in GII in the form of a table, can now be expressed analytically as follows:—

$$\frac{\partial k}{\partial C} = -\frac{2}{3}k, \quad (7)$$

$$\frac{\partial k}{\partial E} = (12 - M_0) \frac{e^{-\frac{2}{3}E}(\frac{2}{3}E + 1) - 1}{E^2}. \quad (8)$$

From (8) and (3) we get

$$\frac{\partial k}{\partial r_0} = \frac{12 - M_0}{a_s} \frac{e^{-\frac{2}{3}a_s r_0}(\frac{2}{3}a_s r_0 + 1) - 1}{r_0^2}. \quad (9)$$

Formula (7) corresponds to changes in the rows of the table in GII, and formula (8) to those in the columns.

(It may be mentioned that by using the photographic absolute magnitude, $M_{pg} = M_{pv} + C + \text{const.}$, in the above formulæ, we get $\kappa_{pg} = \kappa_{pv} + 1$ and $k_{pg} = k_{pv} + 1$, as already found in the earlier papers; the derivations corresponding to formulæ (7) to (9) are unchanged.)

Having thus determined the effect of selective absorption on the observation of absolute magnitudes, we are now able to deduce the changes to which the luminosity function is subject on account of this influence.

3. *Luminosity Function.*—Although it is uncertain whether any form of a luminosity function is valid throughout the whole galaxy, some assumption of at least a limited constancy (such as for a given galactic latitude) is made in every investigation of stellar distribution because there is so far no other way of solution. In the following investigation it is assumed to be constant within the region discussed. There is, however, an essential difference from former investigations, which are based on the knowledge of true absolute magnitudes (or their equivalents), since the present one is based on absolute magnitudes derived from colour indices and therefore falsified by selective absorption. A luminosity function, applied to such kind of knowledge, must therefore change with changing colour excess (or distance). What was called “falsified mixture of spectral types” in GII turns out to be the apparent luminosity function which can now be derived from the constant true luminosity function.

Stars of true absolute magnitude M_0 are observed as of magnitude M_1 (due to falsified colour indices). If the true luminosity function is $\phi_0(M_0)$ then we are led to a new (apparent) luminosity curve $\phi_1(M_0)$ if we apply to the true absolute magnitude M_0 the frequency which belongs, in the true luminosity function, to the absolute magnitude M_1 concerned. Expressing this analytically we have

$$\phi_1(M_0) = \int_0^{r_0} \frac{\partial \phi_0(M_0)}{\partial r_0} dr_0 = \int_0^{r_0} \frac{\partial \phi_0(M_0)}{\partial M_0} \frac{dM_0}{dr_0} dr_0 = \phi_0(M_1). \quad (10)$$

This does not mean, however, that the luminosity function can be shifted unchanged parallel to the M co-ordinate; $M_1 - M_0$ is not constant for a given colour excess or distance, as was pointed out in the previous section. It decreases with increasing M_0 . Also such luminosity functions as have been suggested indicate that the rate of increase of the luminosity function becomes, on the average, smaller with increasing M_0 . Combining these two relations we find that the increase in frequency, $(\phi_1 - \phi_0)$, of the earlier stars (or the stars with smaller M_0) is proportionally larger than that of the later stars so that at any space element, subjected to selective absorption, there will apparently be proportionally more earlier than later stars, in comparison with the true luminosity function. This has been shown, in section 5 of GII, to be not only the deduced but also observed result in a special case.

All this is valid for any luminosity function so far proposed * and for the range of absolute magnitudes corresponding to the limits between which the analytical $M-C$ curve is valid, *i.e.* between $M_0 = -1.5$ and $+8.3$. It can, however, be extended to stars of $M_0 < -1.5$ because here the effect is still more pronounced than in the analytical curve itself. Such an extension at the other end of the curve is not possible, but this difficulty is not very important because the very late stars disappear, on account of their faintness, before any appreciable amount of absorption can affect them.

Formula (10) may be applied to two different types of luminosity function. If we use, for instance, the numerical method, developed by Bok (p. 26), the procedure would be to fill the squares of his $(m, \log \pi)$ table (Table 9) with entries got by multiplying the volume by the apparent frequency instead of the true one. The apparent frequency is easily derived for every distance and absolute magnitude if the selective absorption coefficient is known (formulae (3), (5) and (10)).

In the case of Kapteyn and van Rhijn's † analytical function we are justified by the fact that up to absolute magnitude 8 this luminosity function does not differ too much from the observed values ‡; this magnitude is also our limit and, moreover, faint enough for all stars to be accounted for beyond a certain comparatively small distance. It has been shown that only for stars with absolute magnitudes larger than 9 does their analytical function depart appreciably from the observed results. This function,

$$\phi_0(M_0) = e^{\alpha + \beta M_0 + \gamma M_0^2}, \quad (11)$$

subjected to changes on account of absorption corresponding to formula (10), gives

$$\phi_1(M_0) = e^{\alpha + \beta M_1 + \gamma M_1^2} \quad (12)$$

as the apparent luminosity function. A comparison of the apparent function with the true one

$$\frac{\phi_1(M_0)}{\phi_0(M_0)} = e^{(M_1 - M_0)[\beta + \gamma(M_1 + M_0)]} \quad (13)$$

confirms the above statement that the proportional increase of frequency is larger for earlier stars than for later ones if we realize that β is positive and γ negative. $(M_1 - M_0)$ was shown to decrease with increasing M_0 while $(M_0 + M_1)$ increases with increasing M_0 . Both changes contribute to the decrease of the numerical value of (13) with increasing M_0 , for a given space element. The decrease is given by

$$\frac{\partial}{\partial M_0} \left(\frac{\phi_1(M_0)}{\phi_0(M_0)} \right) = - \frac{\phi_1(M_0)}{\phi_0(M_0)} (\beta + 2\gamma M_0) \quad (14)$$

which, on account of the numerical values of $\beta = +0.5308$ and $\gamma = -0.0345$, remains negative up to absolute magnitude 7.7 or to spectral class K4, *i.e.* to the maximum of this luminosity curve.

There is another qualification that has to be considered. The preceding deductions, being true for the main sequence, do not cover the giant stars present. The absolute magnitudes are determined from colour indices and the latter do not distinguish between giants and main sequence stars. Thus, even when no absorption is present, the distances are falsified because the giants appear to be much nearer than the main sequence stars of the same colour index and at the same true distance.

To correct this influence we have to adjust the luminosity function. We cannot easily apply a correction to an analytical luminosity function, but it is easy to correct the values in a tabular function where the values for $M_0 = 0$ to 2 would have to be decreased proportionally to the presence of giants (which is about 1 to 2 per cent. of the

* See Bok's monograph, p. 18, 1937.

† *Ap. J.*, 52, 31, 1920.

‡ *Groningen Pub.*, 38, fig. 1, 1925; 47, table 6, 1936.

main sequence stars of $M_0 = 5$ to 8 according to published data), while the values for $M_0 = 5$ to 8 would have to be increased by the same numerical amount (a negligible percentage for the higher values).

4. *Density Function*.—Solving a fundamental equation of star numbers, such as

$$A(m) = \omega \int_0^\infty r_0^2 D(r_0) \phi_0(M_0) dr_0, \quad (15)$$

where $A(m)$ is the number of stars of apparent magnitude m and ω a constant determining the size of the area, we get the density function $D(r_0)$. A numerical solution was given by Bok and we have to decide how far we are justified in using this method in our case. Bok's solution is based on the $(m, \log \pi)$ table, mentioned before, which gives the luminosity distribution at progressing distances; it assumes that, at a given distance, the densities of different absolute magnitudes are equal, *i.e.* that the density is independent of the spectral type (equivalent to the assumption of a constant luminosity function).

The method also gives a solution when total absorption is assumed; in this case the computed density at a certain distance is related to the unaffected one by a factor depending on the coefficient of total absorption (a), but independent of the absolute magnitude

$$D(r_0)/\Delta(r_0) = e^{3Pr_0}(1 + Pr_0), \quad (16)$$

where $P = \frac{a}{5 \log e}$ (see also Bok's formula 21).

If selective absorption is also assumed to be present, we can still derive a relation between affected and unaffected density, by the use of the absorption modulus b (formula (2)). There are, however, essential differences from Bok's method: while his apparent distance is derived on the assumption of only total absorption (the absolute magnitudes remaining unchanged), the apparent distance of this paper depends also on the selective absorption. The two apparent distances are therefore not the same. (In order to avoid confusion it may be noted that Bok uses the suffix o , in variance with this paper, to indicate the apparent value.) The second difference is the dependence of the absorption modulus b on the absolute magnitude and distance, while there is nothing of this kind in Bok's case.

For a general solution of the problem we consider the fundamental equation, which now appears in the form

$$A(m) = \omega \int_0^\infty r_1^2 D(r_1) \phi_0(M_1) dr_1. \quad (17)$$

The relation between the true density function and the apparent one becomes

$$D(r_0)/D(r_1) = e^{3Br_0}[1 + r_0(B + B'r_0)] = F, \quad (18)$$

where $B = \frac{b}{\log e}$ and $B' = \frac{b}{\log e} \frac{\partial b}{\partial r_0}$. Formula (18) corresponds to formula (16) and the extension of the bracket term is due to the dependence of B on r_0 .

It will also readily be seen that our factor F is not independent of the absolute magnitude, being a function of $b(M)$. The relation between F and M_0 , for a given distance, is derived from the following formulæ:—

$$B = \frac{1}{5 \log e} \left[a_{pv} - (12 - M_0) \frac{1 - e^{-\frac{2}{3}a_s r_0}}{r_0} \right], \quad (19)$$

$$B' = \frac{1}{5 \log e} (12 - M_0) \frac{e^{-\frac{2}{3}a_s r_0} \left(\frac{2}{3}a_s r_0 + 1 \right) - 1}{r_0^2}, \quad (20)$$

$$F = Ge^{HM_0} (1 + M_0 + K), \quad (21)$$

where

$$G = e^{\frac{3}{5 \log e}} [a_{pv} r_0 - 12(1 - e^{-\frac{3}{5} a_s r_0})], \quad H = \frac{3}{5 \log e} (1 - e^{-\frac{3}{5} a_s r_0}), \quad (22)$$

$$\mathcal{J} = \frac{2}{15 \log e} a_s r_0 e^{-\frac{3}{5} a_s r_0}, \quad K = 1 + \frac{r_0}{5 \log e} (a_{pv} - 8 a_s e^{-\frac{3}{5} a_s r_0}). \quad (23)$$

No use can be made of an analytical method to determine the apparent density function from the fundamental equation (17) because even Schwarzschild's method * (the most general one) is valid only when each of the three distribution functions ($A(m)$, $D(r)$ and $\phi(M)$ in our case) depends only on its own argument; we have, however, seen that $\phi_1(M_0)$ depends not only on M_0 but on r_0 as well.

To employ Bok's numerical method we should have to use the apparent luminosity function, expressed by a $(m, \log \pi)$ table in which the entries were treated as mentioned in the previous section. Then each one of these entries would be correlated to an individual density (instead of to constant densities for all entries of a given distance). A solution of this kind will be shown in section 6 after having ascertained how far the method is justified.

5. *Validity.*—The validity of results yielded by the method outlined above rests on the assumptions made.

The failure of the analytical $M-C$ curve to fit the true values at the extreme ends does not imply a lack of validity in using an apparent luminosity function, since this can be determined graphically outside the range for which the analytical curve is true; thus stars earlier than B_3 (or brighter than $M_0 = -1$) can equally well be included in the application. The density function, however, in its dependence on the transformation factor F and consequently on M_0 is strictly applicable only to stars within the range. The very early stars and those later than K would thus have to be omitted although they form an unseparable part of the counted star number $A(m)$.

If we were to extend the analytical line for the earlier stars (as indicated in fig. 1 by a dotted line) and to use the larger absolute magnitudes yielded by it the corresponding F would be too large (an effect which increases with the distance), and the solution would give apparent density functions increasingly too small, starting with $\log r_0 = 3.200$. There would also be no advantage in using the true absolute magnitudes (instead of those obtained from the analytical extension) in formula (21), within this narrow range of the curve, because the constants G , H , $\mathcal{J}$ and K have also to be changed on account of their dependence on the original constants of formula (1), the basis of the analytical curve. We are, however, able to get the true F for this part of the curve by using formula (18). We can determine B , and consequently B' , graphically from the curve, or we may use the table of GII. In the latter case the values of k have to be diminished by 1 because of $k_{gp} = k_{pv} + 1$.

The density function is not appreciably affected by the uncertainty of the luminosity function at the faint stars, as pointed out by Bok (p. 20), and this effect will be only very slightly increased if we treat stars of $M_0 \geq 9$ as not affected by absorption.

6. *Method of Comparing Theory and Observation.*—The observed entities are apparent magnitudes, apparent absolute magnitudes and star numbers. Thus we can prepare an (m, M) table listing the numbers of stars of different M_1 at different m . On the other hand we can also prepare an empty table of the same structure and complete it with the computed star numbers from a solution of fundamental equation (17), based on the deduction of section 4. A comparison of these two tables will provide a test of the accuracy of the assumptions.

In order to complete the empty (m, M) table we must compute the apparent densities,

* *A.N.*, 185, 81, 1910; 190, 362, 1911; see also Dyson, *M.N.*, 73, 334, and Eddington, *M.N.*, 73, 346, 1913.

multiply these by the apparent frequencies taken from a falsified $(m, \log \pi)$ table, and enter the results in their appropriate squares.

The apparent densities are obtained by computing the true densities by Bok's method, which includes a correction for total absorption, and dividing these true densities by the corresponding values of the factor F . To prepare the falsified $(m, \log \pi)$ table we have to find out (either from formula (5) or from fig. 1) the apparent absolute magnitude M_1 which would be determined by observation (on account of selective absorption) instead of the true absolute magnitude M_0 of this table; then we have to take from the adopted luminosity function the frequency corresponding to M_1 , multiply it by the volume corresponding to the distance concerned, and enter the result at its place in the $(m, \log \pi)$ table. In this falsified table the headings remain the same as in the normal table, only the entries being changed. These entries are the apparent frequencies, which have now to be multiplied by the corresponding apparent densities to give the star numbers required. We have finally to enter the resulting star numbers for each M_0 in those squares of the empty (m, M) table which correspond to the apparent absolute magnitudes M_1 .

The entries of the two tables, *i.e.* the table with the observed and that with the computed star numbers, can now be compared and differences will indicate where our assumptions may be wrong. Systematic differences may show whether there is a systematic change in the luminosity function or whether our assumption on the conditions of absorption have to be altered.

Norman Lockyer Observatory,

Sidmouth :

1943 March 11.

OCCULTATIONS OF STARS BY THE MOON OBSERVED AT THE NIZAMIAH OBSERVATORY, HYDERABAD, DURING THE YEAR 1942

(Communicated by the Director)

(Received 1943 June 11)

TABLE I

No.	Star	Mag.	Phase	Date 1942	U.T.		
					h	m	s
1	B.D. $-1^{\circ} 4523$	9.5	D	Jan. 21	13	35	58.8
2	B.D. $-1^{\circ} 4524$	8.7	D	" 21	13	37	32.1
3	25 Arietis	6.5	D	" 24	12	59	02.8
4	25 Arietis	6.5	R	" 24	14	40	29.3
5	B.D. $+4^{\circ} 230$	8.0	D	Feb. 19	13	53	36.7
6	α Tauri	1.1	D	" 23	14	10	43.1
7	α Tauri	1.1	R	" 23	14	58	04.1
8	115 Tauri	5.3	D	" 24	14	50	13.8
9	Bordeaux $+17^{\circ} 5^h 24^m$ No. 4		D	" 24	14	55	12.9
10	5 Cancri	5.9	D	" 27	15	04	57.4
11	B.D. $+7^{\circ} 2375$	8.3	D	Mar. 2	23	32	15.2
12	B.D. $-18^{\circ} 4864$	6.1	R	" 10	20	55	06.9
13	B.D. $-18^{\circ} 4896$	6.4	R	" 10	22	12	51.9
14	Bordeaux $+16^{\circ} 4^h 8^m$ No. 20		D	" 22	13	24	21.4
15	B.D. $+15^{\circ} 607$	6.8	D	" 22	13	26	18.8
16	B.D. $+15^{\circ} 1851$	6.3	D	" 27	17	27	05.4
17	B.D. $+16^{\circ} 657$	7.2	D	Apr. 19	13	52	28.8
18	B.D. $+11^{\circ} 2136$	7.3	D	" 25	17	43	15.5
19	74 Virginis	4.8	D	" 29	17	27	58.3
20	B.D. $+1^{\circ} 2656$	7.1	D	May 25	14	09	44.4
21	B.D. $-7^{\circ} 3748$	6.6	D	Aug. 17	15	19	36.3
22	75 Tauri	5.3	R	Sept. 29	18	09	17.7
23	B.D. $+15^{\circ} 637$	4.8	R	" 29	19	14	25.9
24	B.D. $-19^{\circ} 5372$	8.8	D	Oct. 16	13	55	05.5
25	Hyderabad $-19^{\circ} 53230$		D	" 16	13	58	28.9
26	B.D. $-19^{\circ} 5378$	9.0	D	" 16	14	04	07.6
27	43 Sagittarii	5.0	D	" 16	14	14	01.7
28	Hyderabad $-19^{\circ} 53252$		D	" 16	14	21	47.9
29	B.D. $-19^{\circ} 5386$	8.5	D	" 16	14	25	27.4
30	B.D. $-19^{\circ} 5387$	6.8	D	" 16	14	40	42.4
31	119 Tauri	4.7	R	Nov. 24	16	31	04.0
32	120 Tauri	5.5	R	" 24	17	21	15.4
33	ζ Cancri	5.1	R	" 27	22	53	55.6

Remarks

No. 11. Observed during eclipse of Moon.

" 17. Time slightly uncertain.

" 21. Observed through clouds.

Three observations (Nos. 6, 11 and 17) were made by T. P. Bhaskaran and the rest by M. K. Bappu. The instrument used was the 15-inch visual refractor except in the case of No. 11, which was observed with a 3-inch portable telescope. The corrections to the times by the standard clock were determined from observations with the transit instrument.

The reductions were carried out by M. V. V. Sastri assisted by S. Aravamudan. No correction has been applied to the observed time of occultation to allow for the error in the mean longitude of the Moon.

TABLE II

No.	$\sigma' - \sigma$	$\cos(\rho - \chi)$	$\sin(\rho - \chi)$	No.	$\sigma' - \sigma$	$\cos(\rho - \chi)$	$\sin(\rho - \chi)$
	"				"		
1	+0.4	+0.70	+0.71	17	-0.6	+0.95	-0.31
2	+1.0	+0.98	+0.18	18	-0.2	+0.86	-0.52
3	+0.2	+0.94	+0.35	19	+0.2	+0.85	+0.53
4	+2.4	-1.00	0.00	20	+3.6	+0.34	-0.94
5	+1.2	+1.00	-0.06	21	+1.0	+0.99	+0.14
6	+1.4	+0.47	-0.88	22	+1.3	-0.83	+0.56
7	+1.8	-0.43	-0.90	23	0.0	-0.68	-0.73
8	0.0	+0.98	-0.20	24	+0.6	+0.91	+0.41
9	+6.2	-0.27	-0.96	25	+1.9	+0.99	+0.17
10	+2.3	+0.32	-0.95	26	+2.5	+1.00	+0.09
11	+3.1	+0.50	+0.86	27	-0.4	+0.90	+0.44
12	-0.8	-0.92	+0.40	28	+0.6	+0.95	-0.30
13	-4.9	-0.42	+0.91	29	+0.1	+0.95	-0.31
14	+5.9	+0.42	+0.91	30	-1.2	+0.77	+0.64
15	-0.2	+0.84	+0.54	31	-0.2	-0.98	+0.19
16	0.0	+0.91	+0.41	32	+0.2	-0.94	-0.35
				33	-0.1	-0.88	+0.47

The mean places for 1942.0 of eleven stars in the list are given in the following table. For star No. 5, proper motion has been applied to the co-ordinates given in the catalogue in computing the mean places for use in the reductions. The positions for the remaining stars were taken from the *Nautical Almanac*.

TABLE III

No.	R.A. (1942.0)	Dec. (1942.0)	Remarks
	h m s	° ' "	
1	0 01 14.16	- 1 16 46.2	Alger -1° 23 ^h 56 ^m .92; Alger -2° 0 ^h 0 ^m .22.
2	0 01 43.17	- 1 22 54.9	A.G. Nicolajew 5948.
5	1 19 22.76	+ 4 56 56.3	A.G. Albany 370; P.M. $\mu_a + 0^s.0006$; $\mu_\delta - 0''.028$.
9	5 22 37.62	+17 41 55.8	Bordeaux +17° 5 ^h 16 ^m .180.
11	10 48 10.64	+ 7 04 16.6	A.G. Leipzig 5610.
14	4 12 58.28	+16 08 40.2	Bordeaux +17° 4 ^h 12 ^m .54.
24	19 13 46.24	-19 05 25.1	Alger 8198.
25	19 13 57.65	-19 08 59.6	Hyderabad -19° 53704; Hyderabad -20° 64787.
26	19 14 07.60	-19 09 55.3	Alger 8203.
28	19 14 34.26	-19 14 52.2	Hyderabad -19° 53730; Hyderabad -20° 64903.
29	19 14 39.95	-19 14 43.2	Alger 8213.

Nizamiah Observatory,
Hyderabad (Deccan), India :
1943 March 20.

OBSERVATIONS OF OCCULTATIONS AT SYDNEY OBSERVATORY
DURING 1942

(Communicated by the Acting Government Astronomer)

(Received 1943 May 13)

The following observations of occultations were made and reduced in duplicate by H. W. Wood (H. W.) and W. H. Robertson (W. R.). The observations were made at an 11½-inch equatorial telescope using a tapping key to record the times on a chronograph, no correction for personal effect being allowed in reading off. The reductions were made by Comrie's method (Supplement to the *Nautical Almanac*, 1938), no adjustment being made to the observed time to allow for error in the Moon's tabular mean longitude.

The observational material is given in Table I (columns 1, 3 and 4 from the *Nautical Almanac*, 1942) and the results of the reductions in Table II (the lunation number is a continuation of the list in the *Nautical Almanac*, 1941). In each case the phase observed was disappearance at the dark limb. In the cases of double stars the two times were meaned for the reduction.

TABLE I

Number	Date	Star	Mag.	U.T.			Observer
				h	m	s	
359	Apr. 24	54 Cancrī	6.3	8	05	54.8	H. W.
371	Apr. 26	ρ Leonis	3.8	7	24	46.8	H. W.
631	July 23	B.D. -14° 43 14	6.2	10	54	22.5	H. W.
708	Aug. 19	B.D. -13° 42 26	6.8	11	49	35.5	W. R.
807	Sept. 17	B.D. -17° 47 73	6.0	9	32	20.0	H. W.
				9	32	25.2	
				8	34	52.6	
815	Sept. 18	B.D. -18° 48 96	6.4	8	34	53.6	W. R.
				10	09	14.9	
838	Sept. 19	ρ Sagittarii	4.0	8	39	07.2	H. W.
965	Oct. 20	λ Aquarii	3.8	10	16	46.8	H. W.
966	Oct. 20	78 Aquarii	6.3	10	06	01.5	H. W.
1079	Nov. 17	B.D. -5° 60 11	7.0	11	11	49.1	W. R.
1166	Dec. 15	B.D. -2° 19	7.3	10	33	50.3	W. R.
1205	Dec. 21	104 Tauri	5.0				

TABLE II

No.	Luna- tion	p	q	p ²	pq	q ²	Δσ	pΔσ	qΔσ	Coefficient of	
										Δa	Δδ
359	239	+ 51	-86	26	-44	74	-0.5	-0.3	+0.4	+ 4.9	-0.94
371	239	+100	- 7	100	- 7	0	+1.8	+1.8	-0.1	+13.9	-0.35
631	242	+100	+ 9	99	+ 9	1	+0.4	+0.4	0.0	+14.4	-0.11
708	243	+100	+ 8	99	+ 8	1	+1.0	+1.0	+0.1	+14.5	-0.13
807	244	+ 98	+22	95	+22	5	+1.8	+1.8	+0.4	+14.3	+0.01
815	244	+ 96	-28	92	-27	8	-1.4	-1.3	+0.4	+13.5	-0.30
838	244	+ 69	+73	47	+50	53	-1.4	-1.0	-1.0	+ 9.1	+0.77
965	245	+100	+ 5	100	+ 5	0	0.0	0.0	0.0	+14.0	+0.34
966	245	+ 83	+56	69	+46	31	-0.8	-0.7	-0.4	+ 9.3	+0.78
1079	246	+ 96	-29	92	-28	8	+0.3	+0.3	-0.1	+14.9	+0.02
1166	247	+ 72	-69	52	-50	48	+0.7	+0.5	-0.5	+13.6	-0.42
1205	247	+ 95	-31	90	-29	10	+1.2	+1.1	-0.4	+14.0	-0.18

Erratum

The President's Address, *M.N.*, 103, No. 2, p. 127, third line from bottom of page,
for ± read P'.

